WORKSHOP ON SET THEORY AND ITS APPLICATIONS (FEB. 19, 2007): ABSTRACTS

Itay Kaplan, *The automorphism tower of a centreless group.*

Given any centerless group G , we can embed G into its automorphism group $\mathrm{Aut}(G)$. Since $\mathrm{Aut}(G)$ is also without center, we can do this again, and again. Thus we can define an increasing continuous series G^α, the *automorphism tower*. The natural question that arises, is whether this process terminates, and when.

I will give historical background, and prove that the process does terminate, even without presence of the axiom of choice. If time permits, I will do more.

Arkady Leiderman, *On Lindelöf $C_p(X)$ spaces.*

We denote by $C_p(X)$ the space of all real-valued functions endowed with the topology of pointwise convergence on X. The following major problems about the Lindelöf property of $C_p(X)$ have been open for many years:

(1) Characterize X for which $C_p(X)$ is Lindelöf.
(2) Assume that $C_p(X)$ is Lindelöf. Does it follow that the product $C_p(X) \times C_p(X)$ is Lindelöf?

It is known that: For any Corson compact X, and for any space X such that X^n is hereditarily separable for each natural n, the countable product $C_p(X)^{\aleph_0}$ is Lindelöf.

In this survey talk we are interested in the results which are related to additional axioms of Set Theory.

Theorem (A. L., V. Malykhin).

(1) Let X be a space with a single non-isolated point. If $C_p(X)$ is Lindelöf then the countable product $C_p(X)^{\aleph_0}$ is Lindelöf.
(2) In the model of ZFC obtained by adding one Cohen real there are two spaces with a single non-isolated point X and Y such that both $C_p(X)$ and $C_p(Y)$ are Lindelöf but the product $C_p(X) \times C_p(Y)$ is not Lindelöf.

Assuming PFA, such a pair of X, Y does not exist (S. Todorcevic).

THEOREM (O. Okunev, K. Tamano). There exist separable, scattered, σ-compact spaces X, Y such that $C_p(X)^{\aleph_0}, C_p(Y)^{\aleph_0}$ are Lindelöf but the product $C_p(X) \times C_p(Y)$ is not Lindelöf.

I'll sketch also recent results of M. Hrusak, P. Szeptycki and A. Tamariz-Mascarua about $C_p(\Psi(\mathcal{A}))$ for the Mrowka space $\Psi(\mathcal{A})$. Here $\mathcal{A}$ denotes a maximal almost disjoint family on ω. Under CH there is $\mathcal{A}$ such that $C_p(\Psi(\mathcal{A}),\{0,1\})$ is Lindelöf.

The talk is intended for a general audience, and all notions will be defined explicitly. If time permits, I'll outline the ideas of some proofs.

Heike Mildenberger, *Menger-bounded subgroups of the Baer-Specker group.*

We investigate necessary and sufficient conditions for the existence of subgroups of the Baer-Specker group whose k-th power is Menger-bounded but whose $(k+1)$st power is not Menger-bounded.

Assaf Rinot, *Nets of spaces having singular density.*

The *weight* of a topological space X is the minimal cardinality of basis $\mathcal{B}$ for X. The *density* of X is the minimal cardinality of a dense subset of X.

If all subsets of X are Lindelöf, and $\mathcal{B}$ is a basis for X, then every open subset of X is the union of countably many members of $\mathcal{B}$.

The theme of our talk is the following problem: Find the least cardinal θ such that there exists a basis $\mathcal{B}$ for X, of cardinality equal to the weight of X, such that every open subset of X is the union of $< \theta$ many members of $\mathcal{B}$.

A *net* for X is a collection $\mathcal{N}$ of subsets of X such that any open set is the union of elements of $\mathcal{N}$. Thus, any basis is a net. For a cardinal θ, define the *relative net-weight with respect to* θ to be the minimal cardinality of a net $\mathcal{N}$ such that any open set is the union of $< \theta$ many elements of $\mathcal{N}$. The main result of this talk is:

THEOREM. Assuming a *very* weak cardinal arithmetic hypothesis. If the density of X is a singular cardinal , λ, then the relative net-weight of X with respect to the cofinality of λ is greater than λ.

In particular, in all currently known models of set theory, if X is a space of density and weight $\aleph_{\omega_1}$, then X is not hereditarily Lindelöf.

Boaz Tsaban, *On a problem of Hurewicz.*

A set of reals X has *Menger's property* (1924) if no continuous image of X in $\mathbb{N}^{\mathbb{N}}$ is cofinal with respect to $\leq^*$. It has the formally stronger *Hurewicz' property* (1925) if every continuous image of X in $\mathbb{N}^{\mathbb{N}}$ is bounded. σ-compactness implies Hurewicz' property, which implies Menger's. Both Menger and Hurewicz conjectured that their property characterizes σ-compactness, and for a long time only consistent counter-examples were known. Hurewicz (1927) also posed the problem whether there is $X \subseteq \mathbb{R}$ which is Hurewicz but not Menger. The problem was raised again by Bukovský and Haleš (2003).

Fremlin-Miller (1988) and then Just-Miller-Scheepers-Szeptycki (19 96) gave a dichotomic existential argument refuting the Conjectures in ZFC. Using the Michael topological technique, Chaber-Pol (2002) improved the dichotomic argument and essentially solved the Hurewicz Problem, alas in an existential manner.

Barotszyński-Tsaban (2002) gave two explicit counter-examples to the conjectures using two specialized constructions. Tsaban-Zdomsky (2005) generalize both constructions and solve the Hurewicz Problem constructively by considering scales with respect to semifilters (collections of infinite subsets of $\mathbb{N}$ closed under almost supersets). Working in $P(\mathbb{N})$ (which is like $\{0,1\}^{\mathbb{N}}$): For each feeble semifilter $\mathcal{F}$ and each $\mathcal{F}$-scale S, all finite powers of $X = S \cup [\mathbb{N}]^{<\aleph_0}$ are Hurewicz and not σ-compact. Viewed appropriately as a subset of $\mathbb{R}$, the field generated by X is Hurewicz, universally null, and universally meager. The Hurewicz problem is solved by using the semifilter $\mathcal{F} = [\mathbb{N}]^{\aleph_0}$, and choosing the $\mathcal{F}$-scale's points such that (the enumerations of) their complements form an unbounded set. To carry this out, descriptive set theoretic properties of semifilters are used.

Lyubomyr Zdomskyy, *Convergence in spaces of continuous functions.*

A topological space Y has the *Pytkeev property* if for each $A \subseteq Y$ and each $y \in \overline{A} \setminus A$, there exist infinite subsets $A_1, A_2, \ldots$ of A such that each neighborhood of y contains some A_n. This is a natural weakening of the *Fréchet-Urysohn* property, in which it is required to have a sequence in A converging to y.

The most well studied case is when $Y = C(X)$, the space of continuous real-valued functions on X, endowed with either the topology of pointwise convergence, or the compact-open topology. In the first case, the Pytkeev property is closely related to infinite-combinatorial notions, and the requirement of $C(X)$ being Pytkeev is very strong. In the second case, however, we show that for each Polish space X, $C(X)$ has the Pytkeev property. All arguments are (essentially) combinatorial.

Necessary definitions will be given in the lecture.

Joint work with Boaz Tsaban.

Boaz Tsaban, Department of Mathematics, The Weizmann Institute of Science, Rehovot 76100, Israel.

E-mail address: boaz.tsaban@weizmann.ac.il