

# Bidders' Strategy for Multi-Attribute Sequential English Auctions with a Deadline<sup>1</sup>

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## ABSTRACT

In this paper we consider a procurement multi-attribute English auction with a deadline. This protocol can be used for agents who try to reach an agreement on an item or issue, which is characterized by several quality attributes in addition to the price. The protocol allows the specification of a deadline since in many real world situations it is essential to conclude a negotiation among agents and to reach an agreement under a strict deadline. Currently, the deadline rules, which are mainly used for auction mechanisms, result in a non-recommended and unstable bidding strategy, i.e., the last minute bidding strategy causes system overhead and inefficient auction outcomes. Therefore, we define another deadline rule, which diminishes the phenomenon of the last-minute bidding-strategy and thus prevents bottlenecks in an agents' network that applies an auction mechanism. We analyzed simultaneous and sequential multi-attribute English auctions with a fixed deadline that can be used for negotiating agents. For each of these protocols combined with the deadline rule, we provide the automated bidder agents with optimal and stable bidding strategies.

## Categories & Subject Descriptors

*Intelligent agents, Multiagent systems.*

## General Terms

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Economics.

## Keywords

Bidding and bargaining agents. Electronic commerce market.

## 1. INTRODUCTION

Negotiating agents can efficiently use auction mechanisms for reaching agreements among agents [8,13,14], and auctions can also be used when the issue under consideration is associated with multi-attributes items [5]. A multi attribute item is characterized by several attributes in which one of them is the price and the others are non-price/quality attributes. For example, in task allocation, the attributes of a deal include the size of a task, its starting time, its ending deadline, the accuracy level, etc. Telephone service providers and Internet portals, as well as video-on-demand suppliers, would like to rent extra storage capacity from suppliers over the Internet. The attributes of the required item in this domain are the storage capacity, the access rates to the data, the availability period and time limit, the level of security, etc. In the International Logistics Supply Chain (ILSC) domain, the required service moves a given cargo from one location to another. The details of this service include arrival time, dispatching time, path length, weight, volume, etc.

In many real world situations it is essential to conclude a negotiation among agents and to reach an agreement within a strict deadline. For example, if an auctioned service in an English auction is to be provided very soon then the negotiation among the agents must be concluded by a defined deadline. Another example can be the case where video-on-demand suppliers need to rent an extra capacity from a supplier over the Internet. In this case, it is obvious that the extra storage capacity is meaningless if it is not provided within the time it is needed. Therefore, a deadline for a negotiation process or an auction mechanism is an essential issue.

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Recently, the fundamental value of a deadline principle in an auction mechanism has been realized by auction house designers and has been applied to many e-commerce applications [10]. There are primarily two types of deadline rules used in auction-houses: (1) the fixed deadline in which an exact time is defined for a strict auction's ending and in which no extensions are allowed; and (2) a deadline, which is based on a predefined interval with no activities from the last proposed bid. In this type of rule, an ending time is defined but the auction is not closed until an interval of time with no activities has passed.

eBay is an example of an auction house that uses a fixed deadline rule. The advantage of this protocol is that the exact time the auction closes is known in advance. On the other hand its disadvantage is that it encourages the last minute bidding strategy, which is not recommended in such environments since it causes system overhead [1,11]. Moreover Roth and Ockenfels [11] proved that "There does not exist a dominant strategy at which each bidder bids his true value at some time  $t < 1$ ". In practical terms, since many bids are submitted at the last moment, the best bid may not reach the auctioneer before the deadline and therefore will not be considered. Sometimes the reason that the best bidder waits until the last minute to bid his true value is to avoid a "bid war" which unnecessarily increases his and the other's following bids [10].

Amazon is an example of an on-line auction house on the Internet that uses the second deadline rule. The advantage of this rule is that it gathers all interested participants at the same time to follow the auction. Its disadvantage is that strategically, the deadline principle does not play any role since the auction continues while there are bidding activities. Also in this auction the bidders are likely to use the last minute bidding strategy [10].

Even though the anomalous behavior of last minute bidding is believed to occur in common value auctions, it has been experimentally shown and theoretically proved that it occurs also in private value auction models [11]. In conclusion, the deadline principle is recommended and needed, though the phenomenon of the last-minute bidding-strategy should be prevented whenever possible.

Therefore, we suggest designing the deadline rules in a manner that provides the bidders with stable strategies and provides the auctioneer with more efficient auction results. The first step of this design is to define the last step/interval/round of the auction in which new bidders will not be permitted to join the auction and yet long enough to ensure that all interested bidders will be able to successfully submit their bids. Each bidder can bid only once in this step. We define two types of bidding in the last phase: *simultaneous* or *sequential*.

This work is an extension of our previous work on multi-attribute English auctions [5,6]. In this paper we propose protocols for multi-attribute auctions with a deadline which are based on a predefined number of rounds. In the sequential version, the bidders bid sequentially in each round according to a predefined order such that each bidder observes the bids of the bidders before him in the round. In the simultaneous version, all the bidders bid in each round without knowing the bids of the other bidder agents. According to the deadline rule we use, the bidder knows exactly when it is his last chance to act. In addition, we propose an automated intelligent agent that uses a sophisticated and steady

bidding strategy thus avoiding application of the non-recommended late-bidding strategy.

For example, we integrate two optional types of practical use: First, the agents can be migrated to work on the auctioneer server. In this case the communication overhead is reduced to local transactions. The disadvantage of this option is that private information of the bidders might be disclosed (unless they apply cryptographic techniques). Second, the bidder agent can be located at the bidder server/computer and send its bid to the auctioneer each time it is its turn to bid. In case of a sequential protocol, each bidder's decision depends on the bidder agent ahead of him and therefore no bottleneck will occur.

Notice that as in each auction protocol with a real deadline rule (i.e., no extensions are allowed), the bidders involve a dimension of uncertainty that leads to speculation. However we tried to diminish the level of uncertainty in the proposed protocols.

In section 2 we describe the model including the bidder agents and the auctioneer agent. We proceed to section 3 in which we provide the automated agents participating in a sequential multi-attribute English auction with stable and optimal strategies. Similarly, in section 4 we consider the simultaneous protocol and provide the optimal bidding strategy. In section 5 we describe related work on the topic of deadline rules and the multi-attribute issue and in section 6 we present our conclusions.

## 2. THE MODEL

The auction model consists of one buyer agent, which plays the role of the auctioneer, and a fixed number of  $n$  seller agents, which are the bidders. The buyer agent that needs a particular item (service or product) initiates the auction process. At the beginning of the auction, the buyer announces its item request, which consists of the item's desired characteristics, and a scoring rule that describes its preferences concerning the item properties. A seller agent, who decides to send a bid, has to specify the full configuration it offers [5,6].

Each buyer agent and each seller agent is characterized by a utility function that describes its preferences. The multi-attribute utility-functions we refer to are based on the Simple Additive Weighting (SAW) method [16]. A utility or a score in the SAW method is obtained by adding the contributions of each attribute. The utility function of the buyer associates a value with each bid, which is the sum of the buyer's level of satisfaction from the various attributes' values. The utility function of each seller associates a utility value reflecting the seller's cost and profit from each bid.

The auctioneer announces a scoring rule at the beginning of the auction. This scoring rule associates a score with each proposed bid and the auction protocol dictates the winner (best scored bid) based on this scoring rule. We assume that each participant knows its utility function, and bidding is not costly. In our model, each seller agent has private information about the costs of improving the quality of the product it sells, or its performance. Each seller agent  $S_i$  (bidder) is assumed to be characterized by a cost parameter  $\theta_i$ , which is its private information. As  $\theta_i$  increases, the cost of the seller to achieve an item of a higher quality also increases, i.e., the seller is "weaker".

Similar to the model described by Che [4], we assume that  $\theta_i$  is independently and identically distributed over  $[\underline{\theta}, \bar{\theta}]$ , where

( $0 < \underline{\theta} < \bar{\theta} < \infty$ ) according to a distribution function  $F$  for which a positive, consciously differentiable density  $f$  exists. Because of complete symmetry among agents, the subscript  $i$  is omitted throughout the rest of the paper.

We analyze a general case of multi-attribute auctions in which there is an arbitrary number of attributes ( $m+1$ ), which is predefined and known to all the participants. One of the attributes is the price ( $p$ ) and the others are quality attributes ( $q_i$  where  $i \in [1, \dots, m]$ ) for which the preferences of the buyer and the sellers conflict. We assume that as  $q_i$  increases, the quality of the item increases. That is, as  $q_i$  increases the cost of the seller to provide it increases since it is harder to provide higher quality items. In addition, the buyer's utility from higher quality items increases. For example, a multi-attribute service of providing a machine can be characterized by three attributes: the price  $p$  of the item,  $q_1$  can denote the speed of the machine and  $q_2$  can denote its accuracy, or the warranty period for this machine. And the seller's cost increases if it provides a more accurate machine, and the buyer's utility is higher if it obtains such a machine

Consider the cost functions of the sellers. We assume that there are fixed coefficients for each of the quality dimensions which are identical for all the sellers. Namely,  $a_1$  is the coefficient of  $q_1$ , and  $a_2$  is the coefficient of  $q_2$  and similarly  $a_i$  is the coefficient of quality attribute  $q_i$ . A seller's cost function, is:

$$C_s(q_1, \dots, q_m, \theta) = \theta \left( \sum_{i=0}^m a_i \cdot q_i \right), \text{ where } a_i > 0. \text{ Based on the cost}$$

function, the seller's utility function is:

$$U_s(q_1, \dots, q_m, \theta) = p - \theta \cdot \left( \sum_{i=0}^m a_i \cdot q_i \right).$$

Notice that, the utility function of the seller is the difference between the price it obtains and the cost of producing the proposed quality values. As the payment it obtains increases, its utility increases.

The above function fits the case where, as  $q_i$  increases the quality of the item or service increases. Thus, higher values of  $q_i$  cause higher costs to the seller and thus, a lower utility for it.

The influence of  $q_i$  is assumed to be independent and linear: as  $q_i$  increases by one unit, the cost of the seller will increase by  $\theta \cdot a_i$ .

It is clear that as  $q_i$  increases, the utility of the buyer increases.

We assume that the  $q_i$ s where  $i \in [1, \dots, m]$  are independent, but not linear: as  $q_i$  increases, the influence of one additional unit of  $q_i$  becomes smaller. This assumption is valid in many domains. For example, enlarging the speed of a machine from 100 Mhz to 200 Mhz will have a higher influence than enlarging the speed from 200 Mhz to 300 Mhz. The effect of  $q_i$  is weighted by  $W_i$ , respectively, where  $W_i$  can be smaller or larger than 1. As  $W_i$  increases, the importance of attribute  $q_i$  to the buyer increases, w.r.t. the price and the other attributes.

We assume that the utility function of the buyer agent (the auctioneer) from an item or service is as follows:

$$U_{buyer}(p, q_1, \dots, q_m) = -p + \sum_{i=1}^m W_i \cdot \sqrt{q_i}, \text{ where } W_i \text{ are the}$$

weights the buyer assigns to  $q_i$ , respectively.

Given the buyer's private utility function, the buyer will announce a scoring rule, which is used for choosing among bids. The scoring rule of the buyer can be different than its real utility function in the sense that the announced weights  $W_i$  may be different than the actual weights  $W_i$ . In particular, the scoring rule

is in the form of:  $S(p, q_1, \dots, q_m) = -p + \sum_{i=1}^m w_i \cdot \sqrt{q_i}$ , where  $W_i$ ,

are the weights that the buyer assigns to  $q_i$ . From the scoring rule we infer that the announced bid's value for the buyer is:

$$V(q_1, \dots, q_m) = \sum_{i=1}^m w_i \cdot \sqrt{q_i}. \text{ The announced values of the weights}$$

$W_i$  can be equal to or different from the real values of the weights  $W_i$ . For example, if  $w_1 < W_1$ , then for some reason, the buyer declares a lower utility derived from each unit of  $q_1$ , than its actual utility from  $q_1$ . Given the publicized scoring rule, each agent interested in selling the item, will join the auction and send a bid describing its suggestion of how to supply the buyer agent's requirements. The bid will be composed of  $m$  quality dimensions  $q_i$  where  $i \in [1, \dots, m]$ , and the price ( $p$ ).

### 3. SEQUENTIAL MULTI-ATTRIBUTE ENGLISH AUCTIONS WITH A DEADLINE

In this section we consider a procurement sequential multi-attribute English auction with a deadline. Before the auction begins the buyer agent announces (1) a scoring-rule function that describes the required item (2) the minimal increment allowed,  $D$  and (3) the maximum number of rounds that will take place till the auction is closed,  $R$ . In addition, each seller is allotted a serial number through a lottery in the beginning of each round.

In each round, each seller can place a bid when its turn arrives. According to the principle of an English auction each placed bid must be better than the previous proposed bid by  $D$  w.r.t. the announced scoring rule. If the seller prefers not to bid then it will not proceed to the next round and it will be considered to have dropped out of the auction.

#### 3.1 Sellers' Strategy in a Sequential English Auction with a Deadline

In a case of the multi-attribute English-auction with a deadline the bidder has to determine the values of all the quality attributes in addition to the price. Similar to the case of a single attribute, one could think that the decision about all the components of the bid should be influenced by the bidder's beliefs about the other competitors and the last proposed bid. However, we proved in [6] as we recall in the following Lemma, that the optimal values of the quality attributes  $q_i(\theta)$  where  $i \in [1, \dots, m]$  are determined based only on the bidder's cost parameter and the announced scoring rule, and independently of the seller's beliefs and the last proposed bid.

**Lemma 1 [6]** Given the scoring rule and the sellers' utility functions, in **multi-attribute** auctions the quality attributes  $q_i$  that maximize the seller's utility are chosen independently of the price and the other sellers' cost parameters, at  $q_i^*(\theta)$  for all  $\theta \in [\underline{\theta}, \bar{\theta}]$ , where,

$$q_i^*(\theta) = \arg \max_{q_i} \{V(q_1, \dots, q_m) - C_s(q_1, \dots, q_m, \theta)\} \quad \text{where} \\ i \in [1, \dots, m].$$

**Sketch of proof:** Since the proof is independent of the auction protocol, this Lemma holds also for the case of sequential multi-attribute English auctions with a deadline assuming the model described in section 2, which also was assumed in [6]. Consequently, the deadline does not affect the choice of the optimal qualities to be offered throughout the auction. ■

Now we proceed by searching for the optimal price and the specific attributes values to be offered in each of the R-1 rounds of the auction. The optimal bid in each of the R-1 rounds is determined according to the feature of the English auction in which the score of the proposed bid should be higher than the score of the last proposed bid by the minimal increment allowed, denoted D. Therefore, similarly to the optimal bidding strategy in English auction without a deadline [6] the optimal bidding strategy in the R-1 rounds of English auction with a deadline is as in Lemma 2. We term the last propose bid *selected*.

**Lemma 2**

Given the scoring rule, the seller's utility functions, and the last proposed bid in a multi-attribute English auction, the seller's best strategy in the first R-1 rounds is to bid the following bid while its utility from the following bid is not negative; otherwise the seller drops out .

$$q_i^*(\theta) = \left( \frac{w_i}{2 \cdot a_i \theta} \right)^2, \text{ where } i \in [1, \dots, m].$$

**Sketch of proof:** In [6] we proved it holds for the case of a sequential multi-attribute English auction without a deadline. However, it holds also in case of an auction with a deadline, since in all the R-1 rounds the bidder only has to signal his participation and to follow the protocol which aims to increment the last proposed bid by the minimal increment required D as in an auction without a deadline. The deadline plays no role in each of the R-1 rounds since all the bidders can see all the proposed bids and consequently can update their bid in the following sessions. In order to understand the intuition lets consider the other two possible strategies. If the seller proposes a price that eventually yields him a score less than the score of the last proposed bid than it will be rejected. On the other hand, if the seller proposes a price that yields him more than the score of the last bid +D then in the next turn he will have to propose more and it actually forces all participants including himself to lose, therefore the seller will propose exactly as claimed. ■

The sellers that survive the first R-1 rounds and reach the last round have to change their strategy in the last round in order to ensure their winning, since they will not have another chance to improve their bid. Thus, each of these sellers should take into consideration the last proposed bid and the number of sellers that

reached the last round, which are located after him in the bidding order. The reason that the seller has to consider only the sellers, which are after him, is that the seller observes the bids of the sellers that proposed ahead of him and their effect on him is concealed in the last proposed bid which is known to him when it is his turn to bid. The seller that is the last to propose among the set of sellers that reaches the last round, should simply wait for his turn and choose a price according to Lemma 2. However, all the other sellers have to speculate and calculate the bid, which yields the maximum expected utility for them.

Assume that seller  $S_j$  reaches the last round and he is not the last to bid. And assume that there are k sellers that may bid after him. Since the qualities are determined according to Lemma 1 and Lemma 2, the question is what should the price of its bid be. Assume that seller  $S_j$  bids  $\{q^*_1, q^*_2, \dots, q^*_m, p\}$  which yields him a score of B. In order to maximize the expected utility from this bid, seller  $S_j$  has to calculate the probability that all the other sellers cannot afford to bid an equivalent bid (w.r.t the scoring rule) plus D. That is, for each of the k sellers that bid after him, the score of their best possible bid must be lower than B. The condition that ensures this is:

$$(1) S(q^*_1(\theta), q^*_2(\theta), \dots, q^*_m(\theta), p^*(\theta)) < B + D \quad \text{for each} \\ l \in [1..k].$$

The next step is to find the best price ( $p^*$ ), with regards to the optimal determined qualities, that seller  $S_j$  can afford. Namely, a bid (combination of price and qualities) that yields seller  $S_j$  a utility of zero according to seller  $S_j$ 's utility function.

$$(2) U_{S_j}(q^*_1(\theta), q^*_2(\theta), \dots, q^*_m(\theta), p^*(\theta)) = p^* - \theta \cdot \sum_{i=1}^m a_i \cdot q^*_i(\theta) = 0 \\ \Rightarrow p^*(\theta) = \theta \cdot \sum_{i=1}^m a_i \cdot q^*_i(\theta)$$

where  $l \in [1..k]$ .

By assigning  $p^*$  and the optimal qualities values as in Lemma 2 in the scoring rule, to the above inequality (1) and by isolating the cost parameter  $\theta_l$  of seller  $S_l$  we obtain the following condition (3) which specifies the case in which seller  $S_j$  may surpass seller  $S_l$  for each  $l \in [1..k]$ .

$$(3) \frac{1}{4(B + D)} \cdot \sum_{i=1}^m \frac{w_i^2}{a_i} < \theta$$

In other words, this constraint means that if seller  $S_l$  is weak, then seller  $S_j$  can beat him. The probability that this situation will occur for a particular  $S_l$  is:

$$(4) \text{Prob}(S_j \text{ overcome } S_l) = 1 - F\left(\frac{1}{4(B + D)} \cdot \sum_{i=1}^m \frac{w_i^2}{a_i}\right)$$

Now the main goal of seller  $S_j$  is to ensure that this situation holds for all the k sellers, which may bid after him in order to win by proposing a bid that yields score B. So the *Expected Utility* (EU) of seller  $S_j$  from bidding  $\{q^*_1, q^*_2, \dots, q^*_m, p\}$  is its utility from bidding  $\{q^*_1, q^*_2, \dots, q^*_m, p\}$  multiplied by the probability that with this bid it will surpass all the k sellers, which may bid after him. That is, the probability to beat one of the k sellers in the power of k. So the EU of a bid  $\{q^*_1, q^*_2, \dots, q^*_m, p\}$  is:

$$(5) EU_{S_j} = \left( p^* - \theta \cdot \sum_{i=1}^m a_i \cdot q_i^*(\theta) \right) \cdot \left( 1 - F \left( \frac{1}{4(B+D)} \cdot \sum_{i=1}^m \frac{w_i^2}{a_i} \right) \right)^k$$

In this phase seller  $S_j$  has to determine a price to bid in the last round, which will maximize his expected utility. By solving a maximization problem of  $EU_{S_j}$  with regard to price  $p^*$ , seller  $S_j$  can calculate the optimal price  $p^*$ . In the following theorem we provide the optimal bid (qualities and the price) to be offered by seller  $S_j$ , which it bids in the last round where there are  $k$  sellers that may bid after him.

### Theorem 1

Given the sequential auction protocol with a fixed deadline, the optimal strategy of the bidder is to bid the minimal required bid in each of the  $R-1$  rounds, and in the last round the bidder should bid

$$q_i^*(\theta) = \left( \frac{w_i}{2 \cdot a_i \theta} \right)^2, \text{ where } i \in [1, \dots, m].$$

and

$$p^* = -s + \frac{1}{4\theta} \sum_{i=1}^m \frac{w_i^2}{a_i}$$

where  $s$  is equal to the following scoring value of the buyer

$$s = \max \{ sol, S(\text{selected}) + D \}$$

where  $sol$  is the score of the optimal bid (including the qualities and the optimal price) and is one of the following values ( $sol_1$ , or  $sol_2$ ) for which the second differentiation of the  $EU_{S_j}$  is negative, where :

$$sol_1 = -D - \frac{f(k-1)}{8 \cdot \theta} + \frac{f}{8 \cdot \theta} \cdot \sqrt{\frac{16 \cdot \bar{\theta} \cdot k \cdot D}{f} + (k-1)^2 + \frac{4 \cdot \bar{\theta} \cdot k}{\theta}}$$

$$sol_2 = -D - \frac{f(k-1)}{8 \cdot \bar{\theta}} - \frac{f}{8 \cdot \bar{\theta}} \cdot \sqrt{\frac{16 \cdot \bar{\theta} \cdot k \cdot D}{f} + (k-1)^2 + \frac{4 \cdot \bar{\theta} \cdot k}{\theta}}$$

$$\text{where : } f = \sum_{i=1}^m \frac{w_i^2}{a_i}$$

### Sketch of proof

The qualities are determined following Lemma 2 and the price is found by solving a maximization problem of the expression  $EU_{S_j}$  in equation (5) above with regard to price  $p^*$ . ■

Because of space restriction, we do not include the proofs here. They can be found in [17].

Empirically, when we checked more than 1000 different parameters' values,  $sol_1$  was always a positive bid while  $sol_2$  was negative. Moreover,  $sol_1$  was the unique maximum point (e.g., Figure 1). In Figure 1 we see that the maximum point of  $sol_1$  is about 14 (14.09). On the negative side, a minimum point  $sol_2$  exists (note that we assume that the score of the bid should be higher than a minimal value, since an undefined interval of the  $EU_{S_j}$  function exists for very low values of  $sol_1$ ).

In the following Lemma we state that as  $\theta$  decreases, the value of  $sol_1$  increases. This is intuitively correct since as the seller is stronger it can afford better bids (with higher score).

### Lemma 3

Given  $k$ , if  $\theta < \bar{\theta}$  then  $sol_1(\theta, k) > sol_1(\bar{\theta}, k)$ .

### Sketch of proof

This can be derived from  $sol_1$ 's expression, given the permitted parameter values. ■

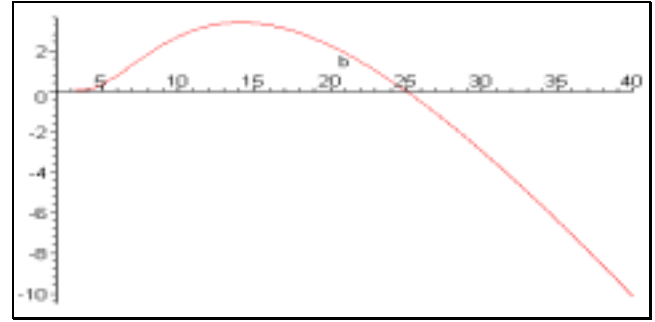


Figure 1. The Seller's expected utility given the scoring value is greater than 2 ( $f=10, D=0.01, k=6, \theta=0.1, \bar{\theta}=1, \underline{\theta}=0$ ).

In Figure 2 we show an example of the optimal bid of a seller in the last round's behavior as a function of the bidder's cost parameter  $\theta$ . That is as the seller efficiency decreases ( $\theta$  increases) the score of the optimal bid decreases because it can only offer worse bids.

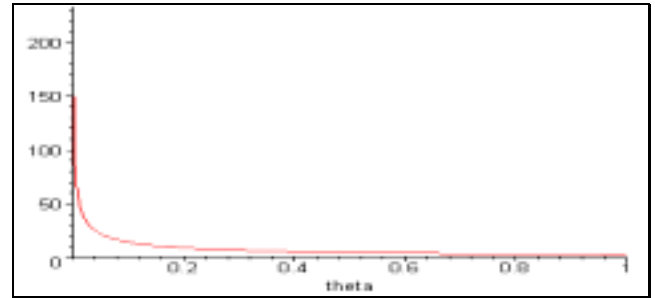


Figure 2: The scoring of the optimal bid, given the bidder's cost parameter.

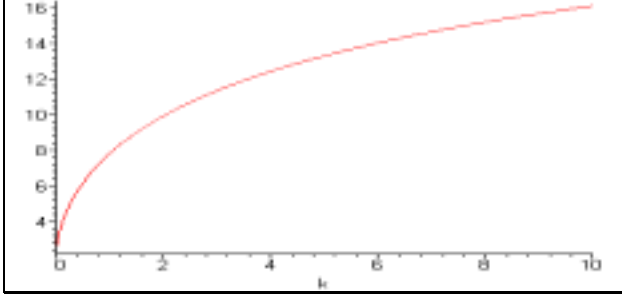
In Lemma 4 we claim that as the number of sellers that may bid after a given seller ( $k$ ) increases, the score of the optimal bid that the seller will propose in the last round also increases. This is demonstrated in Figure 3. The intuition behind this Lemma is that as the competition increases ( $k$  increases), the sellers will propose a better bid which yields the auctioneer a higher score.

### Lemma 4

Given  $\theta$ , if  $k_1 < k_2$ , then  $sol_1(\theta, k_1) < sol_1(\theta, k_2)$ .

### Sketch of proof

This can be derived from  $sol_1$ 's expression, given the permitted parameters' values. ■



**Figure 3: The scoring of the optimal bid of a bidder with a cost parameter  $\theta$  (given the number of bidders after him is  $k$ ).**

To summarize, the optimal strategy of the bidder in the last round depends on its cost parameter,  $\theta$ , and on  $k$ , which is the number of bidders that may bid after him in the last round. The other parameters' values are known to all participants and to the auctioneer.

### 3.2 Buyer's Strategy in a Sequential Multi-attribute English Auction with a Deadline

In this section we would like to reveal the expected revenue of the auctioneer, given the above protocol, and given the optimal bid of each bidder.

Denote by  $sol(\theta, k)$  the best bid of a seller with a private cost parameter  $\theta$ , where there are  $k$  bidders that may bid after him in the last round. Assuming that the real weights of the various attributes are specified in the scoring rule, then the real utility of the auctioneer from a bid that yields score  $sol(\theta, k)$  is equal to  $sol(\theta, k)$ . In other words,  $U_{buyer}(\theta, k) = sol(\theta, k)$  where  $k > 0$ .

Denote by  $ER_{buyer}(\theta, k)$  the expected revenue of the buyer (auctioneer) given that the best seller is characterized by cost parameter  $\theta$ , and there are  $k$  bidders that may bid after him in the last round. In the following lemma, we show how to evaluate  $ER_{buyer}(\theta, k)$ .

#### Lemma 5

Given that the strongest seller is associated with cost parameter  $\theta$  and that the second strongest seller is associated with cost parameter  $\theta_2$ , if  $k=0$ , then  $sol(\theta_2, 1) < ER_{buyer}(\theta, k) < sol(\theta_2, n-1) + (n-1)*D$ . If  $k > 0$ ,  $sol(\theta, k) < ER_{buyer}(\theta, k) < sol(\theta, k) + k * D$ .

#### Sketch of proof

If  $k=0$ , i.e., the strongest bidder is the last one to bid in the last round, then it will simply bid the previous bid+ $D$ . In this case, if the cost parameter of the second best bidder is  $\theta_2$ , the previous bid will be between  $sol(\theta_2, 1) + D$  (if the second best bidder is immediately before the best bidder), and  $sol(\theta_2, n-1) + D*(n-1)$  (if the second best bidder is the first one, and each bidder after it can increase by  $D$ ).

If  $k > 0$ , given that the strongest seller is associated with the cost parameter  $\theta$  indicated by  $S_\theta$ , no seller after him will have an optimal bid which is higher than  $sol(\theta, k)$ . However, there may be

sellers after  $S_\theta$  that will be able to suggest bids better than  $sol(\theta, k)$  since  $S_\theta$ 's speculation may have been wrong. In particular, the revenue of the buyer is  $sol(\theta, k)$  if no seller can suggest more than  $sol(\theta, k)$ . And the buyer's revenue is  $sol(\theta, k) + D$  if one seller that can suggest at least  $sol(\theta, k) + D$  exists. Similarly, it will be  $sol(\theta, k) + 2D$  if there is a seller that can suggest  $sol(\theta, k) + D$ , and there is another seller after it that can suggest  $sol(\theta, k) + 2D$ , and so on.

■

In future work, we intend to develop this formula, and find the explicit value of  $ER_{buyer}$ . In the following lemma, we show the interval of the expected revenue of the buyer, without knowing what the place of the strongest seller in the order of the last round is. Define the *difference between two sellers* to be the difference between their best possible scores. That is, the difference between  $S_i$  and  $S_j$  is:

$$[sol(\theta_i, k) - sol(\theta_j, k)]$$

#### Lemma 6

Suppose that the best bidder has a cost parameter  $\theta_1$ , and the second strongest has a cost parameter  $\theta_2$ . If the difference between them is at least  $D$ , then  $ER_{buyer}(\theta_1, \theta_2)$  is between:

$$sol(\theta_2, 1) + D < ER_{buyer}(\theta_1, \theta_2) < sol(\theta_1, n-1) + (n-1)*D.$$

#### Sketch of proof

Given that the strongest seller has a cost parameter  $\theta_1$ , no seller after  $\theta_1$  will have an optimal bid higher than  $sol(\theta_1, k)$ . However, there may be sellers weaker than the seller with  $\theta_1$  that will be able to suggest bids better than  $sol(\theta_1, k)$ . The strongest seller can be the first bidder in the last round (bidder 1). In this case, the revenue of the buyer will be between  $sol(\theta_1, n-1)$  and  $sol(\theta_1, k) + (k-1)*D$ . On the other hand, it may also be the last bidder (bidder  $n$ ). In this case it will bid the score of the last previous bid+ $D$ . The score of the best bid at time  $n-1$  depends on the best bid in the  $n-1$  rounds. This bid depends on the second best cost parameter,  $\theta_2$ . In particular, it is between  $sol(\theta_2, 1)$  (if the turn of  $\theta_2$  is at time  $n-1$ ) and  $sol(\theta_2, (n-1)) + (n-2)*D$  (if the turn of  $\theta_2$  is at time 1, and all the other bidders are strong enough). Since  $sol(\theta_1, n-1) > sol(\theta_1, 1)$ , and  $sol(\theta_2, 1) < sol(\theta_1, 1)$  we can conclude that  $sol(\theta_2, 1) + D \leq ER_{buyer}(\theta_1, \theta_2) \leq sol(\theta_1, n-1) + (n-1)*D$ . ■

In future work, we intend to find the explicit value of the expected revenue, and compare it to the revenues when using the simultaneous protocols in the last round, in order to reveal which protocol will be preferred by the auctioneer. Moreover, we intend to find out the optimal scoring rule to be publicized by the auctioneer when using this protocol.

## 4. SIMULTANEOUS MULTI-ATTRIBUTE ENGLISH AUCTIONS WITH A DEADLINE

In this section we consider a simultaneous multi-attribute English auction. According to this protocol, the auctioneer defines the number of rounds that will take place until the end of the auction. In each round, all the bidders bid simultaneously, the winning bid

is chosen, and in the next round, each bid should exceed the winning bid of the previous session. In each session, no bidder observes the other bids. Formally, before the auction starts the buyer agent announces (1) a scoring-rule function that describes the required item (2) the minimal increment required,  $D$ , between bids, and (3) the maximum number of rounds that will take place until the auction is closed,  $R$ . The winner is required to provide its bid.

The following Lemma considers the optimal bidding strategy in simultaneous multi-attribute English auctions with a deadline.

#### **Lemma 7**

*The optimal bidding strategy in a simultaneous multi-attribute English auction with a fixed deadline is to bid the minimum possible bid in each round, and in the last round, to bid according to the optimal bidding strategy as in the first-score sealed-bid protocol.*

#### **Sketch of proof**

The bidder is motivated to signal its participation in the auction just before the last round. In the last round, the seller has no information about the other sellers, but it knows that the winning bid will be chosen and implemented and it knows at the beginning of the round how many bidders will participate in the round. Thus, the last round is equivalent to the first price sealed bid protocol (for the case of multi-attribute it is equivalent to the first score sealed bid auction). Intuitively, the bidder is motivated to speculate about the other bidders since it will pay what it bids and if it believes it is stronger than the other participants, and then it can gain more if it bids less than its true value. The bidder will also have no additional information about the other bidders, since in the previous rounds each bidder suggests the minimal possible increment. ■

Since we show that the strategies in the last round are equivalent to the first price sealed bid auction, we refer the readers to [5], where we analyzed optimal strategies for the bidders and the auctioneer in a first price sealed bid auction for multi attribute items.

## **5. RELATED WORK**

### **5.1 Related Work on Deadline Rules**

A deadline rule is a very important property in negotiation processes. For example, both eBay and Amazon auction houses use deadline rules. Although they use different deadline rules, they apply the same basic protocol. According to the eBay and Amazon protocol, at any time each bidder can bid the “current price”, which is the second best bid proposed plus the minimal increment allowed. In each step all the bidders see the second best bid plus the increment. The winner is the bidder that places the last bid and it pays the second highest bid plus the minimal increment.

Roth and Ockenfels [11] consider private value eBay style auctions. They proved that an equilibrium of undominated strategies can exist in which the optimal bidding strategy is to bid the minimal bid at the beginning, and then to wait till the last minute to bid their true value, unless the other bidders deviate from this strategy. In this case, they propose to bid the true value before the last minute. Notice that they assume that in the case of last minute bidding there is a probability less than 11 that the bid

will be successfully transmitted and arrive at the auction house. However, in the case of the Amazon style auction Roth and Ockenfels proved that the optimal bidding strategy is to bid the true value at the beginning of the auction and that there is no need for multiple bidding. The disadvantage of the eBay deadline rule is that it encourages last minute bidding, which is an undesired property. However, the Amazon’s deadline rule discourages users from participating since the auction continues while there are bidders that can bid. This process deters potential bidders. We propose a protocol that is stable and prevents such properties. In [10] they analyzed the reasons that cause the late bidding strategy. For example, in a common value model expert bidders are motivated to wait until the last minute to hide their precious information about the object value.

Easley and Tenorio [7] also investigate the bidding property in on-line auctions. They built a formal model that explains the phenomena of jump bidding. Jump bidding means that the bidder submits a higher bid than necessary. This property also yields an unstable bidding strategy. They explained that this could be a result of cost bidding. According to our protocol there is no incentive to use this strategy.

### **5.2 Related Work on Multi-Attribute Auctions**

In this subsection we present an overview of the research that has been conducted on multi-attribute auctions. For space limitation reasons we omit some primary work.

Guo [9] developed an on-line single-attribute auction mechanism for the case of one seller and many competing buyers using the Internet infrastructure. The auction mechanism it considers assumes that the bids arrive over time and the seller has to make an instant decision of either accepting a bid and closing the auction or rejecting it and moving on to the next proposed bid. Given this mechanism, Guo developed an on-line algorithm that maximizes the seller’s revenue by choosing the best bid in an instant decision. His work differs from our work since we consider a reverse multi-attribute English auction in which the buyer is the auctioneer and the sellers are the bidders. According to Guo’s auction mechanism the auctioneer can terminate the auction at any time, but in our protocol termination will take place according to the announced deadline, which is predefined. The auctioneer decides about the number of rounds such that it assumes that all the interested and potential bidders will have a chance to join the auction.

Another difference is that we assume that any proposed bid is better or equal to the previous proposed bid and in Guo’s protocol this assumption does not hold since the auctioneer hides the received bids and the choice of rejecting or accepting a bid is up to the auctioneer. The auctioneer could lose by rejecting a given bid if it would not receive such bid again.

Bichler [2] made an experimental analysis of multi-attribute auctions. He found that the utility scores achieved in multi-attribute auctions were significantly higher than those of single attribute auctions. The single-attribute auction he refers to is actually a multi-attribute auction in which all the attribute values are fixed and the bidders actually compete for only one-dimension bids.

Very little theoretical work has been done on multi-attribute auctions. Che [4] considers an auction protocol where a bid is composed of a price and a quality. In his paper, he proposed a

design for first score and second score sealed bid auctions, which are based on the announced scoring rule. Also, we consider the English auction protocol, which was not taken into account by Che [4]. Che did not consider deadlines. He considered only one shot sealed bid auctions.

Branco [3] extended the work of Che by assuming that the costs of the firms/bidders are correlated. He considers a governmental procurement auction in which the main goal is to maximize the virtual welfare, which takes into account the private rents that will be given to the firms. Branco uses a method similar to Che's to design the optimal auction considering his model assumptions. In contrast to Branco's work, we assume that the costs of the bidders are independent. In addition, we design the optimal auction from the buyer's point of view and not from the point of view of the overall market's welfare.

Vulkan and Jennings [15] discuss a reverse multi-attribute English auction particularly within the domain of business process management. According to their mechanism and strategies the sellers (bidders) have to recalculate the bid proposed in each step. However, we showed that only the price attribute should be adjusted in each bidding step. Under Vulkan and Jennings' assumptions, they showed that telling the truth about the buyer's (auctioneer's) preferences is an optimal strategy from the buyer's point of view. We, however, developed the optimal auction's design mechanism so that given all the environment parameter values, the buyer can calculate the optimal preferences to announce in order to maximize its expected revenue.

Parkes [12] also considers a multi-attribute auction. In particular, he proposes a family of iterative-based multi-attribute auction mechanisms, for reverse auctions. Even though it seems very similar to our work there are some substantial differences. First, his main goal is to maximize the efficiency and surplus of every seller and the buyer, while our major aim in this paper is to provided the sellers with a strategy that maximizes their utility.

## 6. CONCLUSION

In many real world situations it is essential to conclude a negotiation among agents and to reach an agreement under a strict deadline. Moreover, in many cases it is necessary to conduct negotiation on multiple attributes of an agreement. Currently, the deadline rules which are mainly used for auction mechanisms cause undesired bidding behaviors and inefficient auction results. Precisely, the bidders are encouraged to use the non-recommended "last-minute bidding" strategy that results in system overhead. In addition, most of the automated auctions consider models in which the price is the unique strategic dimension.

Consequently, we define a deadline rule which diminishes the phenomenon of the last-minute bidding strategy and thus prevents bottlenecks in the agents networks. In particular, we consider a procurement multi-attribute English auction with a deadline. We propose an automated intelligent bidder and auctioneer agents that use sophisticated and a stable bidding strategy in order to avoid the late bidding strategy.

In future work we intend to reveal the optimal scoring rule for this auction mechanism and to compare it with the optimal scoring rules used for the multi-attribute auction with no deadlines.

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