Algorithms 2
Exercise 3

1. **Multiple Knapsack Problem**

   Consider the following version of the Knapsack problem, called the *Multiple Knapsack Problem* (MKP for short):

   **GIVEN:** A set $S$ of $n$ items with value $v_i$ and weight $w_i$, and $k$ different knapsacks with capacities $B_1, \ldots, B_k$.

   **OUTPUT:** Find a subset of $S$ with maximum profit (value) which can be packed into the $k$ knapsacks.

   (Assume $P \neq NP$)

   (a) Provide a pseudo-polynomial time algorithm which solves the MKP for $k = 2$ in $O(nB_1B_2)$ time.

   (b) Extend your algorithm for general $k$. Provide running time and space.

   (c) For the MKP with $k = 2$, prove that there does not exist a pseudo-polynomial time algorithm which is polynomial in $n$ and $V = \sum_i v_i$ (HINT: recall that the partition problem defined below is NP-Hard).

   (d) Prove that the existence of a pseudo-polynomial time algorithm does not imply the existence of an FPTAS.

   **PARTITION:**

   **GIVEN:** A multi-set $S$ of $n$ integers.

   **OUTPUT:** Can $S$ be partitioned into two multi-sets $S_1$ and $S_2$ such that $\sum_{s \in S_1} s = \sum_{s \in S_2} s$.

   **REMARK** - The problem is NP-Hard.