Algorithms 2  
Exercise 5

1. **LRU for k-server**  
   Consider the following strategy for the k-servers problem which was defined in class. Given a request, we chose the server who has not answered a request for the longest time to serve the new request. In other words, we chose the least recently used server. Is this strategy $c$-competitive for some constant $c$? Prove your answer.

2. **Paging**  
   Consider the following strategy for the paging problem which was defined in class. Given a request to a page which is not currently in the cache, we chose to remove the page which was requested the smallest number of times (if a page was requested 5 times, replaced, and then requested again 7 times, we say it was requested for a total of 12 times rather than only the last 7 times). Is this strategy $c$-competitive for some constant $c$? Prove your answer.

3. **On-line Batteries**  
   Given an infinite set of machines $\{M_i\}_{i=1}^\infty$ each with a one hour battery, we wish to assign a sequence of tasks $T = (t_1 \cdots t_n)$ to the machines, where $t_i$ is the time needed to execute task $i$ (assume $0 \leq t_i \leq 1$ is a fraction of an hour). However, the tasks $t_i$ arrive on-line; when task $t_i$ arrives, tasks $t_{i+1} \cdots t_n$ are not yet known. The goal is to assign a machine for each task as it arrives so that the number of used machines is minimized, and the sum of all of the tasks that are assigned to each machine doesn’t exceed 1. Provide an $\alpha$-competitive strategy for this problem, for a constant $\alpha$ (should be the smallest constant you think you can achieve).

4. **Donkey Searching**  
   A Bar-Ilan Professor parked his donkey in the university parking lot but, upon his return from class forgot in which side of the parking lot he put the donkey.
   
   If he starts walking in the correct direction he will find the donkey in $d$ steps, but if he walks in the wrong side, the donkey will be found in $n - d$ steps.
   
   a) Describe a competitive on-line algorithm that allows the professor to find his donkey in time $O(\text{cd})$.
   
   b) What is the constant $c$ in your algorithm? Justify.

The figure below describes the professor’s problem

![Diagram of the professor’s problem](image-url)