HUMBOLDT-UNIVERSITÄT ZU BERLIN

Make Some ROOM for the Zeros

Data Sparsity in Secure Distributed Machine Learning

Phillipp Schoppmann, Adrià Gascón, Mariana Raykova, Benny Pinkas

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Data Sparsity

Data Sparsity

- Netflix dataset: 480k users, 17k movies, but only 100M out of 8.5B potential ratings.
  $< 1.2\%$
- Genomics: 3.2B base pairs, but a typical genome differs only at 5M sites.
  $< 0.2\%$
- 20 Newsgroups dataset: 9k vectors, 10^5 features, but only 100 non-zeros per vector.
  $< 0.1\%$

Sparse Storage Formats

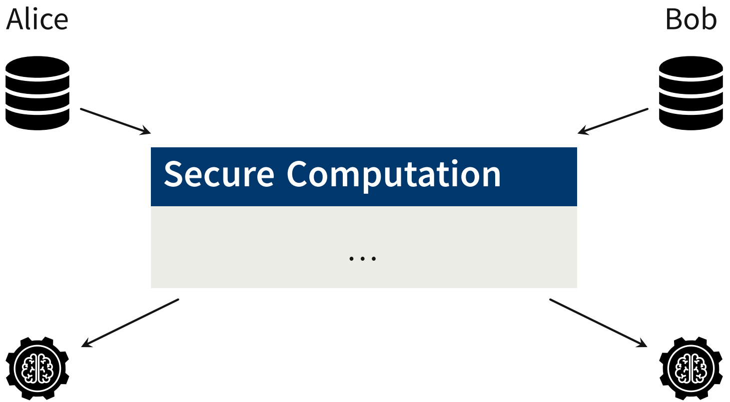
Let R be an arbitrary ring, $\mathbf{b} \in R^d$ be a vector, $\mathbf{A} \in R^{n \times d}$ a matrix.

- Sparse vector: $\textsc{Sparse}(\mathbf{b}) := \big((i, b_i)\big)_{b_i \neq 0}$
- Sparse matrix: $\textsc{Sparse}(\mathbf{A}) := \big(\textsc{Sparse}(\mathbf{a}_i)\big)_{i \in [n]}$

Related to the *Compressed Sparse Row (CSR)* or *Yale format* that is used in scientific computing libraries such as Eigen, SciPy, …

Secure Distributed Machine Learning

Two-Party Machine Learning



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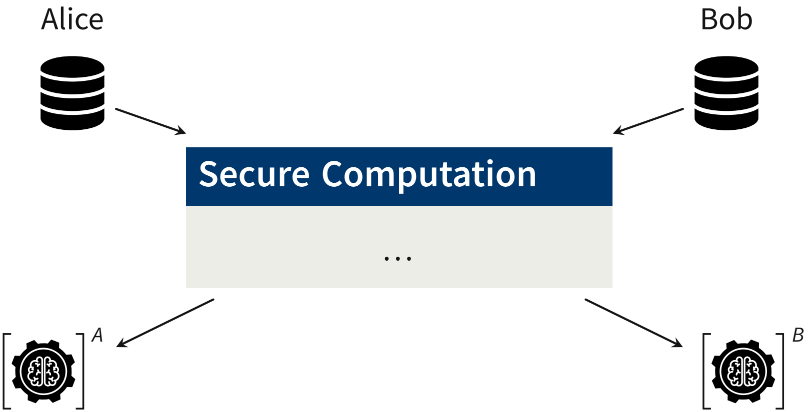

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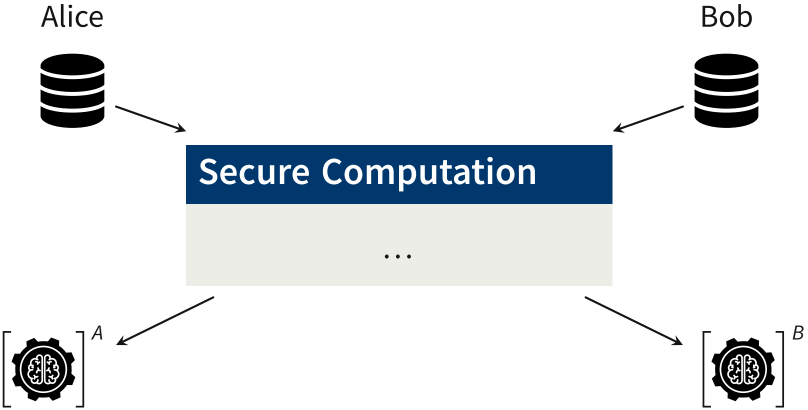
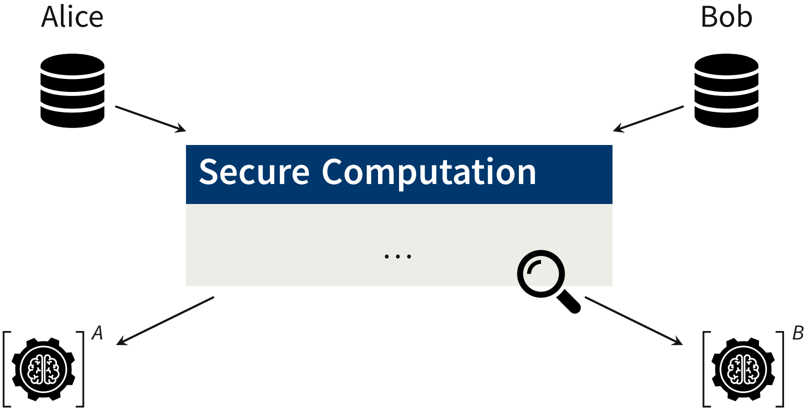


Throughout this talk: Two parties, semi-honest security

Two-Party Machine Learning



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Building Block: Matrix-Vector Multiplication

$\mathbf{A} \in R^{n \times d}$

$\mathbf{b} \in R^{d}$

Secure Computation

Choose random $[\mathbf{c}]^A$, $[\mathbf{c}]^B$, such that

$$[\mathbf{c}]^A + [\mathbf{c}]^B = \mathbf{Ab}$$

$[\mathbf{c}]^A$

$[\mathbf{c}]^B$

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Our Approach

1. Encode sparse vector in a *Read-Only Oblivious Map (ROOM)* data structure.
2. Implement matrix-vector multiplication as a batched oblivious map access.

Basic Primitive: ROOM

Read-Only Oblivious Maps

Query key
q

Database D
(k_1, v_1)
(k_2, v_2)
$\vdots$
(k_l, v_l)

Default value
$\bar{v}$

ROOM Protocol

Compute w as

$$w = \begin{cases} v & \text{if } (q, v) \in D \\ \bar{v} & \text{otherwise.} \end{cases}$$

$[w]^C$

Secret-shared
value

$[w]^S$

Read-Only Oblivious Maps (2)

Query keys

q_1
q_2
$\vdots$
q_k

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(k_2, v_2)
$\vdots$
(k_l, v_l)

Default values

$\bar{v}_1$
$\bar{v}_2$
$\vdots$
$\bar{v}_k$

ROOM Protocol

Compute $\mathbf{w}$ s. t. $\forall j \in [k]$:

$$w_j = \begin{cases} v & \text{if } (q_j, v) \in D \\ \bar{v}_j & \text{otherwise.} \end{cases}$$

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Secret-shared values

$[\mathbf{w}]^S$

Related Functionalities

	PIR [Cho+95]	Batched PIR [Ang+18]	Keyword PIR [CGN98]	Symmetric PIR [Ger+00]	Labeled PSI [Che+18]
Query privacy	✓	✓	✓	✓	✓
Database privacy	×	×	×	✓	✓
Sparse database	×	×	✓	×	✓
Batched queries	×	✓	×	×	✓
Shared Output	×	×	×	×	×

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Shared Output	×	×	×	×	×

ROOM is shorter than *batched symmetric keyword PIR with shared output.* ☺

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Building a ROOM

Naive approach: Ignore database sparsity.

1. Server extends database with dummy elements to span entire key domain:

$$\mathbf{x} = (\perp, \ldots, \perp, \underset{\substack{\uparrow \\ \text{index } k_1}}{v_1}, \perp, \ldots, \perp, \underset{\substack{\uparrow \\ \text{index } k_2}}{v_2}, \ldots)$$

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2. Server encrypts $\mathbf{x}$ element-wise and sends it to client:

$$\tilde{\mathbf{x}} = (\mathrm{Enc}_K(x_1), \ldots, \mathrm{Enc}_K(x_d))$$

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2. Server encrypts **x** element-wise and sends it to client:

$$\tilde{\mathbf{x}} = (\mathrm{Enc}_K(x_1), \ldots, \mathrm{Enc}_K(x_d))$$

3. For each query q_i, Client selects $\tilde{x}_{q_i}$ and the parties perform an MPC with inputs $\tilde{x}_{q_i}$, K, $\bar{v}_i$. The MPC
   a) Decrypts $x_{q_i} = \mathrm{Dec}_K(\tilde{x_{q_i}})$,
   b) Secret-shares x_{q_i} if $x_{q_i} \neq \bot$, otherwise $\bar{v}_i$.

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Communication linear in the key domain!

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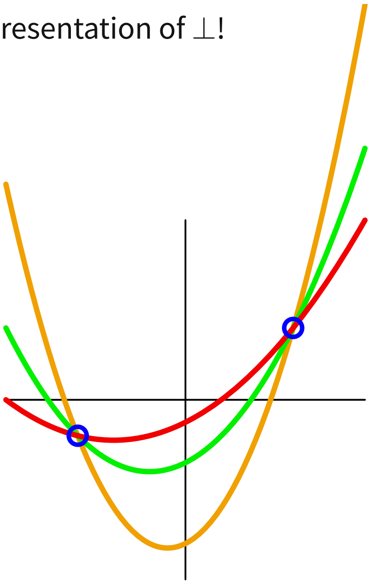

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1. Server pads and encrypts each value in the database:

$$\tilde{\mathbf{v}} = \left(\mathrm{Enc}_K\left(v_1 || 0^s\right), \ldots, \mathrm{Enc}_K\left(v_l || 0^s\right)\right)$$

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   a) Decrypts $x_{q_i} = \mathrm{Dec}_K(\tilde{x_{q_i}})$,
   b) Secret-shares v if $x_{q_i} = (v||0^s)$, otherwise $\bar{v}_i$.

Sparse Inner Product from ROOM

Let $\textsc{Sparse}(\mathbf{a}) := \big((i, a_i)\big)_{a_i \neq 0}$, $\textsc{Sparse}(\mathbf{b}) := \big((j, b_j)\big)_{b_j \neq 0}$.

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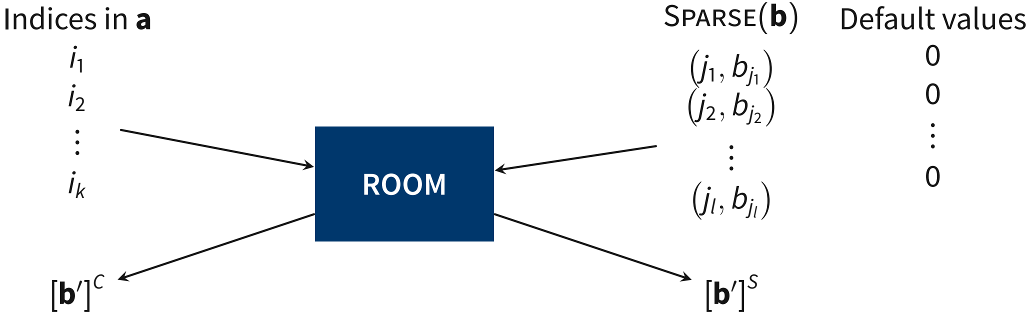

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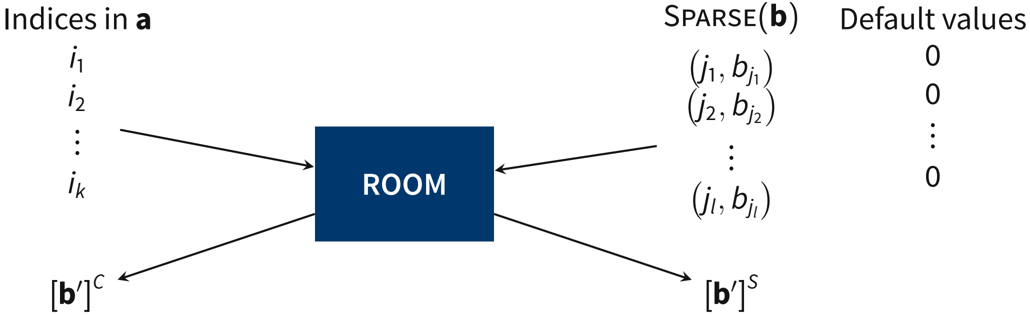
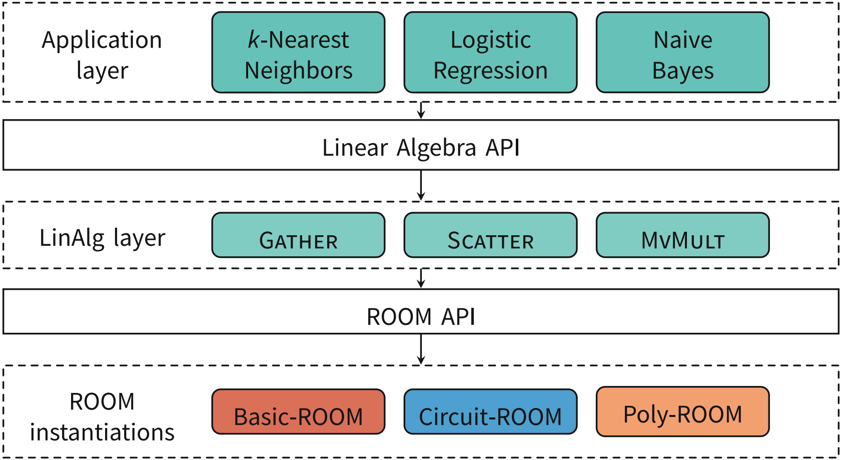


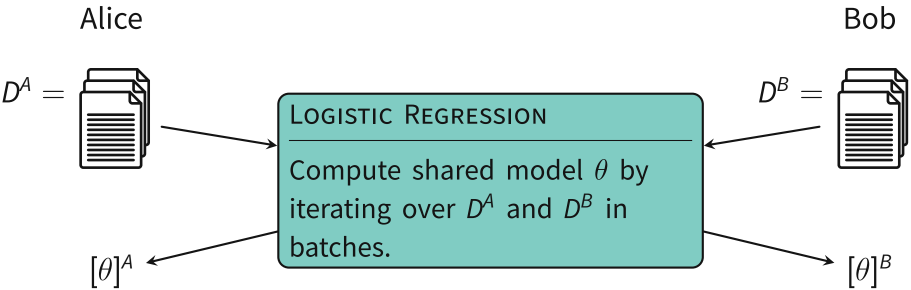
Now, $\mathbf{b}' = (b_i)_{a_i \neq 0}$. Let $\mathbf{a}' = (a_i)_{a_i \neq 0}$. Then $\mathbf{ab} = \mathbf{a}'\mathbf{b}'$.

Applications

ROOM Framework



Logistic Regression



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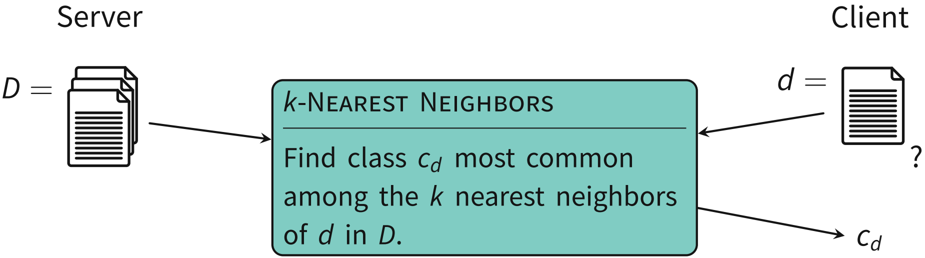
Logistic Regression: Time and Communication

Dataset	*Total Time*		*Communication*	
	SecureML	*Ours*	*SecureML*	*Ours*
Movies	6h29m28.37s	**2h43m46.09s**	4.8 TiB	**187.42 GiB**
Newsgroups	1h42m38.14s	**42m37.68s**	1.26 TiB	**47.63 GiB**
Languages, ngrams=1	**5.9s**	29.89s	790.9 MiB	**500.61 MiB**
Languages, ngrams=2	1h3m7.12s	**6m17.51s**	797.85 GiB	**3.69 GiB**

k-Nearest Neighbors



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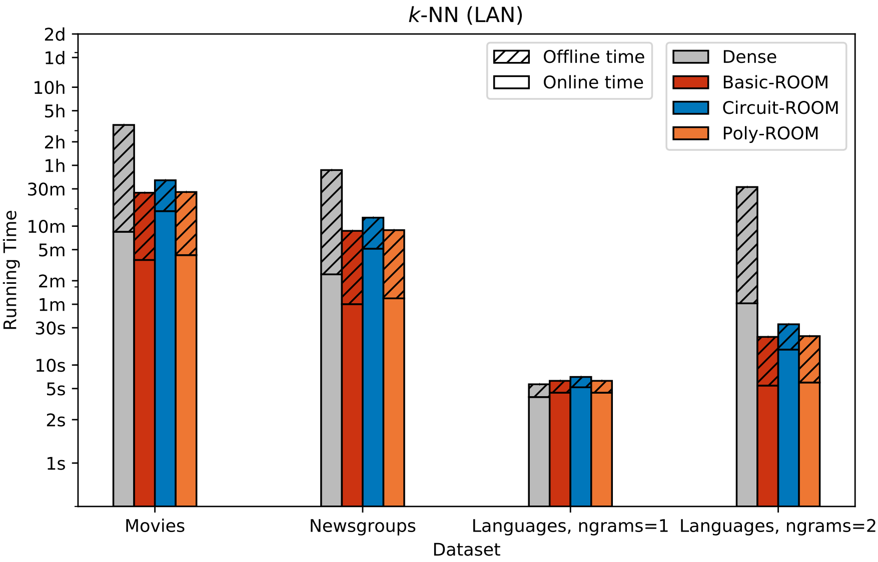
k-Nearest Neighbors: Time



Conclusion

- To scale secure machine learning, we have to exploit characteristics in the *setting* and the *data*.

- We show that for *data sparsity*, using a dedicated data structure helps speed up multiple applications.

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- We show that for *data sparsity*, using a dedicated data structure helps speed up multiple applications.

- Future directions:
  - Improve access times: LowMC, Cuckoo Hashing.
  - Adapt other primitives, e.g. Labeled PSI, flavors of PIR.

References I

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Source Code: https://github.com/schoppmp/room-framework.