Problem Set 1

1. Decremental single source shortest paths.
   Assume that you are given as a black box a data structure $A$ which is capable to maintain a single source shortest paths tree in weighted (non-negative) directed graphs and to answer queries on the distance from the source to any other vertex of the graph. Present an algorithm that given a weighted directed graph $G = (V, E)$ computes its distance matrix only by using the data structure $A$.

2. Decremental reachability in acyclic graphs.
   Let $G = (V, E)$ be a directed acyclic graph and let $v \in V$. A reachability tree of $v$ is a tree that contains paths to all the vertices that are reachable from $v$ in $G$. Present a data structure for maintaining a decremental reachability tree from $v$ whose total update time is $O(|E|)$.

3. Decremental reachability in general graphs.
   Let $G = (V, E)$ be a directed graph (that might contain cycles). Present a data structure for maintaining a decremental reachability tree from $v \in V$ whose total update time is $O(|E| \cdot |V|)$. 