All-Pairs of Shortest paths

1. Let $G(V,E)$ be a directed unweighted graph with $n$ nodes. Show that in $O(n^w \log n)$ time it is possible to compute a matrix $C = (c_{ij})$ such that $\delta(i,j) \leq c_{ij} \leq (1+\epsilon)\delta(i,j)$, for any $\epsilon > 0$.

2. The diameter of a graph is the longest shortest path in the graph. Let $G(V,E)$ be a directed unweighted graph with $n$ nodes. Show that it is possible to find the diameter of $G$ in $O(n^w \log^2 n)$ time.

3. Let $A$ and $B$ be two $n \times n$ boolean matrices. Show how to find witnesses to all entries in $AB$ that have only a single witness. What is the running time of your algorithm.

4. Let $A$ be the the adjacency matrix of an unweighted undirected graph. Suppose that it is possible to compute in $M(n) + O(n^2)$ time a boolean matrix such that entry $(i,j)$ is 1 if and only if the distance between $i$ and $j$ is odd. Show that it is possible to compute the distance matrix of the graph represented by $A$ in $M(n) \log n + O(n^2 \log n)$ time.

5. Show how to implement Seidel’s algorithm using only boolean matrix multiplication.