Resolving Uncertainty: A Unified Overview of Rabbinic Methods

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Introduction

The Talmud frequently discusses situations in which a legal decision must be made in the face of uncertainty with regard to the facts. Such discussions invoke a number of different principles but at least some of these principles are not given explicit formal definition and the relationships between them are not spelled out. In this paper, I will formally define each of the variety of Rabbinic methods used to resolve (or dispel) empirical uncertainty and will place them in the context of certain now well-understood probabilistic concepts. In this way, I hope to present a unified overview of Rabbinic laws concerning uncertainty.

The modern theory of probability is twice removed from Rabbinic laws concerning uncertainty. First, in its current form the theory of probability is simply the study of a particular class of functions and is not concerned with assigning probabilities to real-world events. Second, even if on the basis of certain stipulations the theory is applied to actual events, it remains descriptive and not prescriptive. Nevertheless, certain philosophical issues which have arisen as a result of attempts to explicate the meanings of probabilistic statements are highly relevant to a proper understanding of Rabbinic approaches to uncertainty. I will use ideas taken from the study of foundations of probability where these ideas seem helpful but will try to refrain from belaboring the analogy for its own sake.

One historical point needs to be emphasized. The modern theory of probability has its roots in the work of Pascal and others in the 17th century. It would be utterly anachronistic to attribute to Tannaim and Amoraim any foreknowledge of these developments. Moreover, doing so does not purchase any explanatory power with regard to Rabbinic approaches to uncertainty. At the same time, the claim that the ancients were bereft of any systematic thinking with regard to uncertainty is both arrogant and demonstrably false. I will use modern ideas about the foundations of probability as a starting point for identifying which probabilistic insights do and do not lie at the root of Rabbinic pronouncements on such matters.

Nevertheless, my approach in this article is unabashedly ahistorical: rather than chart a chronological progression of ideas or identify conflicting schools of thought, I will attempt to harmonize a broad range of sources. Where a Tannaitic or Amoraic source permits multiple interpretations, I will not outline all views but rather select the most straightforward or consensual interpretation. Likewise, I will relate to the central ideas discussed in the vast post-Talmudic literature devoted to Rabbinic laws concerning uncertainty but, for the sake of offering as straightforward and unified a treatment as possible, I will cite opinions of the commentators in an extremely selective manner[1]. The fact that I marshal the support of a particular commentator regarding a particular point should in no way be taken to mean that I can claim such support regarding related points.

In the first part of this article, I will use the distinction between *ruba d'itta kaman* and *ruba d'leyta kaman* to motivate a discussion of distinct definitions of probability. This will lay the groundwork for the explication of a number of thorny Rabbinic concepts involving uncertainty and indeterminacy.

Interpretations of Probability

The gemara in Hullin 11a-11b interprets the verse "*acharei rabim l'hatot*" to mean that decisions of a beit din are decided by majority (to be precise, a majority of two is required to convict). This is then generalized to a principle of *ruba d'itta kaman* (henceforth: *RDIK*; literally: a majority which is in front of us) which includes other cases such as that of "nine stores", i.e., a piece of meat is found in the street and all that is known is that it comes from one of ten stores, nine of which sell kosher meat.

[1] This literature includes a number of classical book-length treatments, such as *Shev Shmaatsa*, *Shaarei Torah* and *Shaarei Yosher*. In addition, *Pnei Yehoshuah* often makes reference to an unpublished monograph he wrote on this topic called *Klal Gadol*. Some recent contributions to the classical literature that are especially noteworthy can be found in the writings of R. Osher Weiss. See especially his *Minchas Osher on Bereishit*, Chapter 58 and Shiur on Parshat Mishpatim (5763, transcript distributed by Machon Minchas Osher). An important contemporary book that treats this topic from a historical perspective and covers a number of sources discussed in this paper in detail is Nachum L. Rabinovitch, *Probability and Statistical Inference in Ancient and Medieval Jewish Literature* [Univ. of Toronto, 1973]. Contemporary articles devoted to probabilistic aspects of Rabbinic methods for handling uncertainty are too numerous to list. Three articles which I have found exceptionally useful appeared in *Higayon* (Vol. 4): L. Moscovitz, "On the principles of majority (*rov*) and *itchazeq issura*" (in Hebrew); N. Taylor "The definition of *rov* in halakhah" (in Hebrew); Y. Werblowsky, "*Rov* and probability". My gratitude to all these authors. Two important papers by J. Beck and V. Shtern, "The talmudic concepts of *safek* and *sfek sfeka*" and "The talmudic concept of *zil batar ruba*", which I became aware of only as this article was completed, appeared in *Otam beMoadam* (Bulletin of the Young Israel of Brookline), Nissan 5761 and Tishrei 5762, respectively. Finally, I am grateful to a good number of friends and colleagues who made very helpful comments on earlier drafts of this article. I am not listing them by name only for fear of inadvertent omission.

In such cases we apply the principle that *kol d'parish me-ruba parish* (henceforth: *parish*; literally: that which is removed, was removed from the majority). The gemara states that this inference covers only the principle of *RDIK*, of which Sanhedrin and "nine stores" are offered as typical examples, but not *ruba d'leyta kaman* (henceforth: *RDLK*; literally: a majority which is not in front of us). The gemara offers a number of examples of *RDLK* where the majority is followed because it would be impossible to perform mitsvot or adjudicate cases without doing so (but concludes that precisely because of that impossibility these cases can't serve as a basis from which to infer a general principle of *RDLK*). Several cases of *RDLK* that are illustrative are that the husband of one's mother (at the time of conception) may be presumed to be one's father, that a child may be presumed to be potentially fertile and that a murder victim may be presumed not to have been suffering from a prior life-threatening condition.

What is the difference between *RDIK* and *RDLK*? Although the names are suggestive, the gemara offers no explicit definition of *RDIK* and *RDLK* and no rationale for treating them differently. We might, however, shed considerable light on the distinction by considering an interesting philosophical debate dating back to the 1920's which covers similar conceptual territory. The rest of this section will consist of a slightly lengthy diversion through that territory.

Let's consider carefully what exactly we mean when we say that the probability of some event is p/q. Early (17th and 18th century) work in probability was motivated to a large extent by games of chance (coins, cards, dice). Thus when somebody said that "the probability of the event H is p/q" it was understood that what was meant was that the event H obtained in p out of q equally likely possible outcomes. Thus, for example, when we say the probability that the sum of two throws of a die will be exactly six is 5/36, we mean that there are 36 equally likely possible throws and 5 of them have the desired property. Similarly, in the case of the found meat, there are ten possible sources for the meat and nine of them are kosher, so we might say that the probability that the meat is kosher is 9/10. This understanding of probabilistic statements is usually called the "classical" interpretation.

What is interesting for our purposes is that the classical interpretation turns out to be inadequate as a definition of probability. This became obvious once insurance companies began using probability theory to compute actuarial tables. What does it mean to say that "the probability that a healthy forty-year-old man will live to the age of 70 is p/q"? What are the q equally likely possible outcomes, p of which find our insuree celebrating his seventieth birthday? No such thing. This led philosophers such as Reichenbach and von Mises[2] to suggest the "frequentist" interpretation of probability: the statement that "the probability that a healthy forty-year-old man will live to the age of 70 is p/q" means that of the potentially infinite class of hypothetical healthy forty-year-old men, the proportion who will see seventy is p/q.

It is important to understand that according to each of these interpretations, the classical and the frequentist, there is always some subjective aspect in assigning a probability to an event. In the case of classical probability, this subjective element is rather benign: we need to define the underlying "equally likely" cases, or what is called in formal parlance, the "sample space". For example, in the case of "nine stores", we might just as plausibly use as our sample space the three shopping malls in which the stores are concentrated or perhaps the ten thousand pieces of meat that are unequally distributed among the stores. The choice of which sample space is most appropriate is ultimately a matter that must simply be stipulated. It is tempting to imagine that the "right" sample space is the one in which the various elements are equally probable. But obviously this formulation is circular since it is the very notion of probability that we are trying to define. To be sure, in many cases, there is a rather obvious first choice of sample space. For example, in tossing a die, we would naturally identify the six possible faces as our sample space. This intuition rests on some sort of "indifference principle" (why should one face be more likely than another?). But such indifference principles have proved remarkably resistant to precise formulation. Ultimately, the assignment of sample space is a matter of stipulation.

If in the case of classical probability, assigning a probability to an event requires a bit of judgment, in the case of frequentist probability such an assignment is fraught with judgment. Think of the example in which we wish to determine the probability that a particular child is potentially fertile (actually in the situation described in the gemara we wish only to determine that this probability is greater than ½). We wish to do so by invoking some rule that says: there is some reference class A in which this child is a member and the expected proportion of members of A which are potentially fertile is p/q. This expected frequency is in turn determined by our past experience with members of class A and the frequency of fertility they exhibited. But what class A is appropriate? Should A be the class of all young mammals or all human children or perhaps the class of all children who share this child's medical history or the class of

[2] See R. von Mises, *Probability, Statistics and Truth* [Dover, 1957], originally published in German [Springer, 1928].

children who share this child's medical history and genetic stock? If we define the class too broadly we run the risk that our experience with the class is irrelevant to the particular child in question. If we define it too narrowly we run the risk that our experience with the class is too limited to provide any reliable information with regard to the class in general. And if we define it bizarrely (say, the class consisting of this child and all major household appliances), the results are, well, bizarre. The selection of the reference class A as well as the determination that our experience with samples from that class is sufficient to project some statistical law onto the whole class are matters of judgment.

Consider now the extreme case of a probabilistic statement such as "the probability that the United States will attack Iraq within two months is 60%". The problem with such statements is that the events in question belong to no natural class since the ensemble of relevant facts renders the case unique. It is implausible that we mean to say that in 60% of cases like this an attack occurs, because there aren't any cases quite "like this". Since according to the frequentist interpretation every probabilistic statement must refer to some class, these statements are utterly meaningless within the frequentist framework and indeed are rejected as such by von Mises and others.

One attempt to salvage such statements as meaningful has involved yet another interpretation of probability, the "subjectivist" interpretation. According to this interpretation, the statement that the probability of some event is p/q is taken to reflect the degree of certainty with which some rational observer is convinced of the correctness of the statement, as might be reflected in a betting strategy. Unlike the previous interpretations, such an interpretation does not require the identification of any relevant class. For example, for someone to say that the probability that the United States will attack Iraq within two months is 60% is simply to say that they regard as fair either side of a bet with 3:2 odds in favor of such an attack occurring.

To summarize, there are at least three different kinds of probabilistic statements: classical, frequentist and subjective[3]. For each type, any instance of such a statement is meaningful only to the extent that at least one potentially fuzzy factor can be plausibly defined. In the classical case this factor is a sample set, in the frequentist case it is a reference class, and in the subjectivist case it is simply the strength of a hunch.

3 To be sure, these three do not constitute an exhaustive list of all interpretations that have been suggested. Other interpretations, such as the "logical" interpretation, purport to subsume one or more of these. Certainly, for our purposes these three will suffice.

In the following sections, we will see how various Rabbinic methods can be best understood in relation to these different types of probabilistic statements. Moreover, we will see that different ways of resolving the fuzzy aspects of probabilistic statements can neatly account for certain apparent anomalies. In the next section, we will explain differences between the conditions and consequences of *RDIK*, on the one hand, and those of *RDLK*, on the other. After that we will clarify when *RDIK* is applied and when a converse rule (*kavua*) is applied and will elucidate the difference between *safek* (uncertainty) and indeterminacy. Finally, we will discuss the mechanics of *sfek sfeka* and contrast it with cases of asymmetric *sfekot* where the asymmetry is ineffective (*ein safek motzi midei vaday*).

Ruba D'Itta Kaman and Ruba D'Leyta Kaman

We will define the principle of *RDIK* more precisely in the next section but for now it is enough to define it roughly as follows: A random object taken from a set a majority of the members of which have property P, may be presumed to have property P. As so defined, the principle does not require any (but perhaps the most naive) probabilistic notions. Nevertheless, it is evident that the classical interpretation is fully adequate for a probabilistic formulation of *RDIK*: *RDIK* amounts to specifying the members of the set as a sample space and following the result with probability greater than ½. Note that *RDIK* refers specifically to a set of q concrete objects, p of which have some property, while the classical definition of probability refers more generally to q possible outcomes (which may be abstract).

The classical interpretation is, however, clearly irrelevant to the examples of *RDLK* we have seen. The frequentist interpretation, on the other hand, squares with *RDLK* perfectly[4]. Simply put, all examples of *RDLK* are statistical laws: most children born to married women are fathered by their husbands, most children are ultimately fertile, most people are not about to die, etc.

The identification of *RDIK* with the classical interpretation and *RDLK* with the frequentist interpretation will help us clear up a number of difficulties[5]. Let us begin

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[4] See the discussion in Rabinovitch (*op.cit.*), Chapter 3.

[5] The case should not be overstated. Certainly the Rabbis regarded majority as relevant for resolving uncertainty and certainly they distinguished between two distinct kinds of majority. While it is true that these notions of majority can be neatly embedded in full-blown theories of numerically quantifiable probability, it certainly does not follow that the Rabbis were in conscious possession of any such theory. Nor, by the way, is this necessarily a bad thing. Although scholars since Leibniz have occasionally toyed

with the question of which is stronger, *RDLK* or *RDIK*. Acharonim have marshaled proofs for each possibility the most salient of which are the following.

The strength of *RDLK* relative to *RDIK* can be clearly seen in the following: It is well-established that we don't convict in capital cases based on mere likelihood (Sanhedrin 38a). Thus, consider the case of an abandoned baby boy, called an *assufi*, whose mother is one of a given large set of women one of whom is a non-Jew. In this case, there is a *RDIK* in favor of the child's Jewish maternity. While such a child may be regarded as a Jew for certain purposes, a woman who eventually marries him cannot be convicted of adultery, "*she-ein horgin al hasafek*" (Rambam *Hil. Issurei Biah* 15:27, see also Makhshirin 2:7, Ketubot 15a). Nevertheless, consider another case of uncertain maternity, in which a woman has a relationship with a child that is typical of that of mother and son but, as is generally the case, there are no witnesses to the birth. In this case, there is a *RDLK* in favor of the woman's maternity. If she and the "son" are witnessed having sexual relations, they can be convicted for incest "*she-soklin vesorfin al hachazakah*" (Kiddushin 80a, Rambam *Hil. Issurei Biah* 1:20). Clearly, *RDLK* in these cases is stronger than *RDIK*.

There are other cases, however, in which *RDIK* appears to be stronger than *RDLK*. For example, R. Meir holds that *chaishinan lemiuta* – a *rov* does not trump a contrary *chazakah d'me-ikara* unless it is an overwhelming *rov* (Yevamot 119b). Thus, for example, dough of *terumah* that was last known to be *tahor* but was found in the proximity of a child who is *tamei* cannot be burned, according to R. Meir, on the basis of a *RDLK* that children typically pick at dough in their vicinity (Kiddushin 80a). Tosafot (Yevamot 67b s.v. *ein chosheshin*, Yevamot 119a s.v. *kegon*; see also Mordechai on Hullin, Perek HaZroah, Para. 737) argues that this principle holds only with regard to *RDLK* but *RDIK* always trumps a *chazakah d'me-ikara*. Moreover, according to R. Yochanan, in the case of the *terumah* dough even the Rabbis who disagree with R. Meir would concede that the *terumah* can't be burned on the basis of this *RDLK*. Nevertheless, they would not so concede in a case of *RDIK* (see Kiddushin 80a, Rashi s.v. *im rov*). Thus, in these cases *RDLK* is weaker than *RDIK*[6].

with the idea of using probability theory to numerically quantify evidence for legal purposes, one is hard-pressed to find even a single instance in which such flights of fancy have advanced the cause of justice. See, for example, the epilogue of J. Franklin, *The Science of Conjecture* [Johns Hopkins Univ., 2001].

[6] It has also been suggested that the principle *ein holkhin bemamon achar harov* holds only with regard to *RDLK* but that *RDIK* does apply even for *dinei mamonot* (Rashbam, Bava Batra 93a s.v. *de-hu gufei*; Terumat haDeshen 314), but it is not clear that this distinction is relevant here. See footnote 9 below.

We might be able to reach a definitive answer regarding which is stronger, *RDIK* or *RDLK*, by explaining away one or the other set of proofs. But to do so would be to answer the wrong question. To understand the crucial difference between *RDIK* and *RDLK*, let's recall the difference between the classical interpretation of probability and the frequentist interpretation.

In the case of classical probability, the part that is left to judgment is rather limited. Typically, a rather straightforward sample space is taken for granted. Once that's taken care of, assigning a probability is a simple matter of calculation. (In fact, in the limited case of *RDIK*, the cases need only be counted.) In the case of frequentist probability, however, selecting a reference class and then estimating frequencies within the class requires a substantial investment of judgment. With what confidence can we assert that for some class A the event in question occurs with some sufficiently high frequency? Answering this question, even loosely, is inevitably a matter of judgment.

The crucial difference, then, between *RDIK* and *RDLK* is that, while *RDIK* is a counting principle that can be applied on an *ad hoc* basis, every *RDLK* is a general statistical law that can only be applied if it has received Rabbinic sanction. Since *RDLK* is always a product of Rabbinic judgment, it stands to reason that this judgment is exercised variably. The apparent contradiction regarding the relative strengths of *RDIK* and *RDLK* simply reflects the fact that different applications of *RDLK* are assigned different strengths (both in terms of the strengths of the laws themselves and in terms of the strength of the evidence for the laws). On the other hand, applications of *RDIK*, which do not – so to speak – pass through Rabbinic hands, are all treated in a uniform manner.

Consequently, if you've seen one *RDIK* you've seen them all. Unless there is some countervailing principle which prevents its application, *RDIK* is a decision procedure which resolves, but does not dispel, uncertainty in favor of the majority regardless of whether p/q is .99 or .51. That is, in applying the principle of *RDIK* we acknowledge that there is uncertainty but the *RDIK* allows us to decide in favor of the majority much in the way that a majority vote settles a case in court. Invoking *RDIK* is not sufficient, however, to achieve the degree of certainty necessary to establish the facts of a capital case.

Unlike *RDIK*, however, there are various types of *RDLK*. There are three types of decision rules and, depending on Rabbinic judgment, *RDLK* can be any one of them.

- The middle type is the one we have seen in the case of *RDIK* – a resolution procedure. These are often referred to as "*hakhra'ah*".[7] An example of an *RDLK* of this type is that most births are not of healthy males (Hullin 77b).
- There are stronger decision rules which simply render irrelevant the minority possibility – some examples of *RDLK* are treated as certainties in the sense that we proceed *as if* the uncertainty has not simply been resolved but rather has been dispelled altogether (or as some would have it, there is no *leidat hasafek*). These are often referred to as "*beirur*". It is about these that we say "*soklin al hachazakot*" – in capital cases certainty is required and these examples of *RDLK*, unlike any example of *RDIK*, do indeed provide certainty for legal purposes[8].
- Finally, there are weaker decision rules which are merely "defaults" in the sense that they are applied only as last-resort tie-breakers when no more substantive decision rule is available. These are often referred to as "*hanhagah*". The typical example of a default rule in halakhah is *chazakah d'me-ikara* (with the possible exception of *chezkat mammon* – possession – which is regarded as a substantive desideratum and not a mere default[9]). In some cases, *RDLK* is established merely as a default rule so that at most it can

[7] The terms I use here – "*hakhra'ah*", "*beirur*" and "*hanhagah*" – are those used in the lengthy discussion in *Shaarei Yosher* 3:1-4. Although the distinctions among these three categories are implicit already in the gemara, explicit conceptualization of them is a relatively recent phenomenon. As a result, there is no consensus with regard to nomenclature. Often different nomenclature is employed, sometimes even with some of these same terms used in different permutations. R. Chaim Volozhiner (cited in *Nachalat Dovid* Resp. 24) notes that the word "*chazakah*" is used in all three ways.

[8] To be sure, in capital cases, the act which is grounds for conviction must be established by direct witnesses and no amount of circumstantial evidence is sufficient (see Sefer HaMitsvot LT 290). Even the strongest type of *RDLK* is applied only towards establishing the necessary background facts (see *Shev Shmaatse* 4:8). It should also be noted that legal certainty is required only with regard to the facts of the case; obviously with regard to the procedural matter of voting for conviction, a procedural majority—i.e. *RDIK* – is sufficient.

[9] This is the most plausible explanation of Shmuel's oft-cited principle *ein holkhin bemamon achar harov.* In fact, though, the gemara applies this principle only twice (Bava Kama 27a and Bava Batra 92b) and both those cases permit a much narrower interpretation of the principle, namely, that typical market behavior is irrelevant for establishing the intentions of the participants in a specific transaction.
Note also that there is at least one case (Ketubot 75b) in which R. Gamliel appears to hold that *chezkat haguf* trumps *chezkat mamon*. *Pnei Yehoshuah* argues that according to this view *chezkat haguf* is also more than a default rule, but other explanations are possible. The case of *chezkat haguf* is in any case interesting in a way that bears mentioning: unlike other forms of *chazakah d'me-ikara*, which are purely normative principles, the principle of *chezkat haguf* is rooted in a particular model of the real world, namely, one in which as many facts as possible persist as long as they are not known to have been contradicted. Researchers in artificial intelligence find such models to be especially useful for many technical reasons. Compare, for example, Y. Shoham, *Reasoning About Change* [MIT Press, 1988], Ch.5 and R. Elchonon Wasserman, *Kovets Shiurim*, Ketubot 75b, Para. 265.

neutralize[10], but not defeat, another default rule such as *chazakah d'me-ikara*. For R. Meir, most cases of *RDLK* are of this variety.

Ruba D'itta Kaman and Kavua

Let's now return to the principle of *RDIK* and attempt to define it more precisely. We have already seen that according to the gemara (Hullin 11a), this principle covers both the case of majority vote in Sanhedrin and that of "nine stores" where the meat is found on the street. Moreover, the gemara often invokes the related, though clearly not identical, principle of *bitul b'rov*: a mixture of permitted and forbidden objects may sometimes be assigned the status of the majority. Although, the gemara does not specify the source of this principle, most commentators follow the opinion of Rashi (Gittin 54b, s.v. *lo yaalu*) that it is derived from "*acharei rabim l'hatot*" as well.

The generalization from the case of majority vote to cases such as "nine stores" is not inevitable - the case of voting is more a procedural issue than one of resolving uncertainty. As R. Elchonon Wasserman (*Kunteres Divrei Soferim* 5:7) puts it: if Eliyahu haNavi declared the questionable piece of meat to have come from the minority we could take his word for it, but if he ruled in accord with the minority position in the Sanhedrin we would ignore him (as in the case of *tanuro shel achnai*, [BM 59b]). Similarly, it has been argued (*Shaarei Yosher* 3:4) that the extension to *bitul b'rov* does not seem, on the face of it, to be one which can be glibly asserted. Clearly, the principle the gemara wishes to base on *acharei rabim l'hatot* is sufficiently general that it covers all of the above cases.

Before we consider what this principle might be, let's consider the remarkably similar situation with regard to another decision principle, namely, *kol kavua k'mechtsah al mechtsah dami* (henceforth: *kavua*; literally: that which is fixed is as half and half). Like *RDIK*, the case identified in the gemara as the "source" case of *kavua* is a procedural matter. Someone throws a stone into an assembly of nine Israelites and one Canaanite, intending to kill whichever person the stone happens to hit. The question is whether this unspecific intention is sufficient intention to kill an Israelite to warrant conviction for murder. The Rabbis apply the principle of *kavua* to determine that the Israelite majority does not render the intention sufficient (Ketubot 15a). What exactly the principle might be requires explanation. But note that in this

[10] To be precise, the duel between such a *rov* and the competing *chazakah d'me-ikara* results in a standoff. See Yebamot 119a and Kiddushin 80a Tosafot s.v. *smokh*. Later we will encounter cases in which a purported *rov* is so dubious that it is simply defeated by a *chazakah d'me-ikara*.

case there is no doubt that the actual victim was indeed an Israelite and not a Canaanite. The issue under discussion is only whether the intention to kill "some member of this group" can be regarded as the intention to kill an Israelite. Thus, there is no uncertainty regarding any of the facts of this case and no decision-method for resolving empirical uncertainty is called for.

The gemara then cites as the classic example of *kavua*, the parallel case to that of "nine stores" that we considered above: "If there are nine stores which sell kosher meat and one which sells non-kosher meat and someone took [meat] from one of them but he doesn't know from which one he took, the meat is forbidden."

The parallelism between *RDIK* and *kavua* is remarkable. In both, the "source" case involves a technical legal question and includes no elements of empirical uncertainty and in both the standard case is a version of "nine stores" in which the central issue is apparently one of uncertainty. This suggests that *RDIK* and *kavua* do not directly concern uncertainty, but rather are dual principles regarding mixed **sets** which cover cases of uncertainty as a by-product.

The principle of *RDIK* might thus be formulated this way:

> Given a set of objects the majority of which have the property P and the rest of which have the property not-P, we may, under certain circumstances, regard the set itself and/or any object in the set as having property P.

The principle of *kavua* is the opposite of this:

> Given a set of objects some of which have the property P and the rest of which have the property not-P, we may, under certain circumstances, regard the set itself, and consequently any object in the set, as being neither P nor not-P but rather a third status. We can call this status *hybrid*, or perhaps, *indeterminate*.

It is important to note that *RDIK* comes in two varieties: *RDIK* can assign a single status to the entire mixed set (as in the case of *bitul*) or it might assign a status directly to an individual object in the set (as in *parish*). *Kavua*, on the other hand, comes in only one variety: a hybrid status must be assigned to a set and then only indirectly to an individual item in the set. When *kavua* is invoked, each individual item in the set loses its individual identity and is regarded simply as a fragment of an irreducibly mixed entity. It is not treated as an individual of *uncertain* status but rather as a part of a set that is *certainly* mixed.

Given this, we are ready to answer the central question: When do we apply *RDIK* and when do we apply *kavua*?

Roughly speaking, the idea is that when an object is being judged in isolation, it must be assigned a status appropriate to an individual object; when it is judged only as part of a set, it can be assigned some new status appropriate for a set. Kavua can only be invoked in the latter case. To see this distinction very starkly, consider two scenarios in each of which we have before us a box containing nine white balls and one black ball.

Scenario 1: I reach into the box, pull out one ball without showing it to you and ask: What is the color of this ball?

Scenario 2: I don't reach into the box, but instead ask: What is the color of a random ball in this box?

In the first case, if you were to answer, say, "black", your answer would be either true or false, but either way would be an appropriate response to the question that was asked. There is a determinate answer to the question, although this answer is unknown to you. In the second case, the answer "black" (or "white") is neither true nor false, since there is no determinate answer to the question. You could say nothing more specific than that the box contains both white and black balls.

Obviously, the case of the stone-thrower considered above is analogous to scenario 2 – asking about the status of an unspecified member of the group is like asking about the color of an unspecified ball. The appropriate level at which to assign status in this case is the level of the set, not the level of the individual, and the set is indeed mixed. This is the sort of case in which *kavua* can be invoked.

By contrast, a piece of meat that is found in the street is clearly analogous to scenario 1 – the status of a particular item is in question. This is the kind of case in which *RDIK* is invoked. Now, admittedly, the case of a piece of meat bought in one of the stores might plausibly be regarded as analogous to scenario 1 since the act of buying could be considered analogous to pulling out a specific ball. However, the Rabbinic principle is, somewhat counter-intuitively, otherwise: apparently, the critical moment is the one *prior* to actually encountering the piece in question. When the piece is found on the street, it is judged as an individual because prior to the moment that it is found, it is already no longer "in the set". When the piece of meat in question is bought in the store, prior to its being bought it is indeed "in the set".

The distinction between *kavua* and *RDIK* might be restated in terms of the issue of sample space selection considered above. *RDIK* assumes the "standard" sample space. In the case of the meat found in the street, that sample space is the set of stores. But *kavua* entails the selection of a non-standard, but entirely sensible, sample space: the single element consisting of the entire set of stores. This single item is mixed.

Let us now spell out in detail the precise method for determining when to apply *RDIK* and when to apply *kavua*.

1. First, there are a number of cases in which *kavua* cannot be invoked because a hybrid status is inappropriate.
   - In the case of a vote in Sanhedrin which is, by definition, a mechanism for rendering a decision.
   - If uncertainty regarding the status of an individual object which belonged to the set arose only after the object had been isolated from the set (*parish*), then it is this object alone which must be assigned some status. While a member of a set consisting of objects some of which are P and some of which are not-P can be assigned a hybrid status *as part of the set*, an individual object being assigned a status on its own cannot. Thus, we need to choose either P or not-P for this object and we choose the majority of the set from which it comes. For example, in the case of "nine stores" in which the meat is found on the street, the isolated piece of meat is assigned either the status 'kosher' or the status 'non-kosher'.
   - Similarly, if the set is somehow "incohesive", so that each object in it is regarded as having left the set, we apply *RDIK* and not *kavua*. Thus, for example, a set of travelers passing through a town do not constitute a set for purposes of *kavua*, while the residents of the town do (Ketubot 15b and compare Yoma 84b)[11].

[11] It is often erroneously thought that mobility (*naidi*) is the converse of *kavua* (in the sense of "stationary"). In fact, mobility is simply one possible symptom of the items in question failing to constitute a cohesive set. See M. Koppel, "Inclusion and Exclusion" (in Hebrew), *Higayon* 1 , pp. 9-11; M. Koppel, "Further comments on *rov* and *kavua*" (in Hebrew), *Higayon* 4, pp. 49-52; L. Moscovitz, "On the principles of majority (*rov*) and *itchazeq issura*" (in Hebrew), *Higayon* 4, pp. 18-48.

- Finally, if it is not certain that the set contains any objects that are, say, not-P, then the set is said not to satisfy the condition of *itchazek issura*. Such a set cannot be assigned a hybrid status and *RDIK* is invoked rather than *kavua*. Thus, the Tosefta (Taharot 6:3) already considers a case in which we are given a mixture of ten loaves, including one loaf that is *tamei*, that is eaten in two rounds of five loaves each. Those who eat in the first round are *tamei* because at that point the set certainly contains one *tamei* loaf, but those in the second round are *tahor* because by then the set might not contain a *tamei* loaf. Similarly, a mixture of objects including one which has been used for idolatry is forbidden for use but if one of the objects is destroyed the mixture is permitted for use (Zevachim 74a as understood by Rambam *Hil. Avodah Zarah* 7:10).[12]

To summarize: in all cases in which we are not assigning a status to a mixed set, *kavua* is not invoked but rather *RDIK*[13]. Note that although in these cases the membership of the doubtful item in the set, or the cohesiveness of the set itself, may be inadequate for invoking *kavua*, this does not diminish the relevance of the set for purposes of *RDIK*. Thus, for example, even though the piece of meat found on the street cannot be assigned a hybrid status because it is not part of the set, the fact that the meat is known to have originated in the set still renders the composition of the set (i.e., the majority) relevant to determining the status of the piece.

2. When the above rule does not apply, so that at issue is the status of a mixed **set**, we first check if the principle of *bitul* can be applied. If *bitul* can be applied, the set is no longer regarded as a mixed set but rather as a uniform set and *kavua* is inapplicable. Thus, for example, a mixed set consisting of one

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[12] For the same reason, if some – but not all – of the objects in the mixture are mixed with other, permitted, objects the resulting mixture is permitted. While this is not the only possible interpretation of the gemara in Zevachim, it is the one offered by the Rambam. Nevertheless, with regard to a seemingly analogous case considered by the gemara, a mixture involving valuable objects, Rambam (*Hil. Maachalot Assurot* 16:10) does not permit the second mixture. It has been speculated that in the latter case, where *bitul* fails due to a set property rather than a property of the forbidden object, every object in the set is regarded as forbidden so that the presence of any such object in the second set is sufficient for *itchazek issura*.

[13] Of course, it is possible in such cases that the set from which the object in question originates includes an equal number of objects that are P as are not-P. Such cases are regarded as *safek*, more about which below.

(non-distinguishable) non-kosher piece of meat and more than one kosher pieces is regarded as a permissible set[14].

3. Finally, there are a number of cases in which we are dealing with a mixed set but *bitul* is not applicable:
    - First, if the objects in the set are each identifiable as either P or not-P (*nikar bimkomo*). For example, in "nine stores" the status of each store is known, it is only the origin of a particular piece of meat that is in doubt. Clearly, in such a case, we can't define the set as either P or as not-P; as a set it is both.
    - Second, if individual objects in the set are each regarded as sufficiently significant that the status of each cannot be subordinated to the status of the set (*chashivi velo beteili*) or if *bitul* is inapplicable for any other reason. Thus, given a herd of oxen including one that has been sentenced to death and is forbidden for use, we can't invoke *bitul* due to the significance of living creatures and hence we invoke *kavua* by default (Zevachim 73b).
    - Third, if the set includes an equal number of objects that are P as are not-P. In such a case, *bitul* is obviously not possible.

In each of these cases,[15] we are dealing with an irreducibly mixed set and the principle of *kavua* is invoked: the set is assigned a new hybrid status (*P and not-P*) as are individual objects drawn from the set.

[14] On this understanding *bitul* applies to the entire set and is roughly analogous to the case of votes in Sanhedrin from which it is (by some accounts) learned. This reading of *bitul* is also neatly parallel with our understanding of *kavua*, which also applies to sets. According to this view, all elements of a mixed set on which *bitul* has been effected (in favor of permissibility) should be permitted even simultaneously and to a single person. In fact, the Rosh (Hullin Ch. 7, Para. 37) so rules; Tosafot Rid (Bava Batra 31b), however, disagrees. It may be that Tosafot Rid holds that *bitul* does not apply to the entire set but rather to each individual object in the set separately, along the lines of *parish*. See *Kovets Shiurim*, Bava Batra, Para. 127.

[15] This reflects the view of Rashba (Hullin 92a) that *kavua* applies any time *bitul* fails. However, Tosafot (Zevachim 73b s.v. *ela*, Gittin 64a s.v. *asur*) holds that the only authentic cases of *kavua* are *nikar bimkomo*. All agree, though, that when the failure of *bitul* is *mi-derabanan*, as it always is in cases like *chashivi*, that *kavua* also holds only *mi-derabanan*.

Kavua and Safek

The crucial distinction between *safek*, which reflects *uncertainty* regarding an individual object, and *kavua*, which is a *definite* hybrid status assigned to a set, cannot be over-emphasized. When *kavua* is invoked, it is the definite mixed status of the entire set that concerns us and not the uncertain status of any individual item in the set. It is generally the failure to appreciate this distinction which leads to the conclusion that *kavua* is completely counter-intuitive.

Let's consider for a moment the alternative, more common, explication of *kavua* as merely a leveling of the playing field in which the case is treated as a symmetric *safek*. On this understanding, which I reject, the sample space would contain two elements: kosher and non-kosher. According to my explanation, in cases of *kavua*, the sample space consists of a single element: the entire mixed set. Might not the phrase *mechtsah al mechtsah* suggest that the rule is in fact that we assign each status a probability of ½, that is, that we have a sample space consisting of *two* elements? Why do I reject this possibility?

First of all, because such a rule would be arbitrary and the one I argue for is perfectly sensible. Moreover, the notion that *mechtsah al mechtsah* refers to a probability of ½ is utterly anachronistic. The assignment of probabilities to the range [0,1], so that ½ is in the middle, is a relatively recent convention. The phrase *mechtsah al mechtsah* certainly refers to set composition and not to probability. Specifically, it refers to the third case in Rule 3 of the *kavua*/*RDIK* rules above in which *kavua* applies to a mixed set that includes an equal number of objects that are P as are not-P. The point of the phrase *kol kavua ke-mechtsah al mechtsah dami* is that in all cases that satisfy the conditions for *kavua*, *RDIK* is not invoked just as it is obviously not invoked in the case where there is no majority.

Finally, there are important halakhic differences between cases which are deemed *safek* and cases where *kavua* is applied. For example, if a person had before him two indistinguishable pieces of meat, one kosher and one non-kosher – a case of *kavua* – and he ate one of them, he is obligated to bring an *asham taluy*. But if he had before him one piece, possibly kosher but possibly non-kosher – a case of *safek* – he is not so obligated (Rambam *Hil. Shegagot* 8:2 based on Kritut 17b)[16]. Similarly, if a mouse

[16] The term used in the gemara is *ikba issura*. Rambam, however, uses the term *kavua*. Nevertheless, one should not extrapolate too freely from *asham taluy* to *kavua* in general. Some cases regarded as *kavua* for purposes of *asham taluy*, would not ordinarily be considered cases of *kavua*. For example, if a *melakhah* is performed possibly on Shabbat possibly on another day, an *asham taluy* may be brought.

takes a piece from a mixed pile of pieces of chamets and of matzah, in a manner such that the principle of *kavua* would apply, into a house which has been inspected for Pesach, the house must be re-inspected. But if it took a single piece of which has an even chance of being chamets or matzah into the house – this is a *safek* – the house need not be re-inspected (Pesachim 9a as understood by Rambam *Hil. Chamets u-Matzah* 2:10-11). In the case of *safek*, we can presume that an inspected house remains free of chamets since one possible resolution of the uncertainty regarding the subsequent events is consistent with this presumption. In the case of *kavua*, however, there is no uncertainty to resolve. Rather, some object of known mixed status has certainly been brought into the house; this is enough to nullify the presumption.

Now that we have established that cases of *kavua* are not cases of *safek*, which cases are in fact *safek*? The status of an object is *safek* when it is not judged as part of a set (so that *kavua* and *bitul* do not apply) *and* it has not been removed from a set with a majority (so that *parish* does not apply)[17] *and* it does not belong to some reference class for which some statistical law is known (so that *RDLK* does not apply). A clean example of *safek* is one in which a piece of meat is found in the street and might have come from one of two stores, one kosher and one non-kosher.

Actually, few cases of *safek* are that neatly symmetric. In the following sections, we will deal with two types of asymmetric *safek*, *safek haragil* and *sfek sfeka*, and see which types of asymmetries matter and which don't.

[17] Recall, though, that *RDIK* resolves uncertainty but does not dispel it. On this basis, Shev Shmaatse (2:15) argues that certain Talmudic references to *safek*, such as *safek asiri* (BM 7a) and *safek mamzer* (Kidushin 73a), include cases of *RDIK*. This claim serves as the starting point for a great deal of "yeshivishe torah" but is not essential for an understanding of the underlying issues.

Ein Safek Motsi Midei Vaday

In several places in the gemara we find the principle *ein safek motsi midei vadai*. For example, in Avodah Zara 41b, Resh Lakish argues that if an idol is found broken we can assume that its owner renounces it and thus it is no longer forbidden to make use of it. R. Yochanan rejects this argument on the grounds that it was certainly (*vadai*) initially an idol but only possibly (*safek*) renounced and thus on the basis of the principle *ein safek motsi midei vadai* we may not use it. Tosafot (s.v. *v'ein safek*; Hullin 10a, s.v. *taval v'alah*) points out that the *safek* referred to in such cases is in fact a "*safek haragil*" – that is, it is more likely than not that the idol was renounced. According to Tosafot, then, at least some cases of *ein safek motsi midei vadai* are actually cases of a majority failing against a *chazakah d'me-ikara*. But then it would appear that these cases contradict the widely accepted rule (against the view of R. Meir) that a majority defeats *chazakah d'me-ikara* (Nidda 18b). Why are these cases treated differently?

We can answer this question by answering a more elementary one: what exactly is a *safek haragil* and why is it not simply called *rov*? R. Osher Weiss (*Minchas Osher on Bereishit*, Chapter 58) suggests that what Tosafot calls a *safek haragil* is specifically a case in which some event does not belong to any recognized reference class covered by a statistical law. In such cases, for which von Mises would argue that the notion of probability is undefined, the principle of *RDLK* does not apply. Rather, the *safek* is *ragil* only in the sense that a typical observer might find one possibility to be subjectively more likely than not. But this subjective probability is not relevant; a *safek haragil* is, for legal purposes, no kind of majority at all[18].

Simply put, the term *safek* refers not to cases where two possibilities are known to be equiprobable, but rather to cases in which two possibilities exist and neither *RDIK* nor *RDLK* are applicable. We might think of a *safek* as a case with a sample space consisting of two opposite elements.

[18] Recall that we saw that *RDLK* comes in at least three varieties (depending on the naturalness of the reference class and the strength of the statistical law), the weakest of which is a simple default rule. We can think of *safek haragil* as a degenerate cases of *RDLK* in which the reference class is not deemed convincing enough to even rate as a default rule. One difference between *RDLK* as a default rule and *safek haragil* is that *RDLK* as default neutralizes a *chazakah d'me-ikara* (Yebamot 119a) while the fact that a *safek* is *safek haragil* is irrelevant in the face of a such a *chazakah*. Note, though, that on at least one occasion (Ketubot 9a s.v. *v'ee ba-it*) Tosafot conflates *RDLK* as default with *safek haragil*. Moreover, according to Tosafot, the fact that a *safek* is *ragil* is not entirely irrelevant: Tosafot (Pesachim 9a, s.v. *v'im timtzi lomar*) holds that *safek haragil* might not be subject to the rule that *safek tumah be-reshut hayachid tamei*.

Sfek Sfeka

Not every case in which both *RDIK* and *RDLK* fail necessarily results in *two* possibilities. Recall that in our comparison of *RDIK* with the classical interpretation of probability above, there was a glaring gap. The sample spaces in *RDIK* were limited to concrete objects in a set. What happens when an object is not drawn from a set of objects in some proportion (so that *RDIK* in the usual sense does not apply), but there are more than two possibilities for assigning its status. Does there exist some extension of *RDIK* to such abstract sample spaces? According to one opinion, *sfek sfeka* is just such an extension.

The principle of *sfek sfeka* is this: If a particular prohibition holds only if both conditions A and B hold, and in fact both A and B are in doubt, then we can assume that the prohibition does not hold. Rashba (Resp. 1:401) states that *sfek sfeka* operates "like a *rov*". There are two main branches of thought regarding the nature of this analogy. According to one approach, *sfek sfeka* is like *rov* in its effect but not in its mechanics. Thus according to Ra'ah (Bedek haBayit on Torat haBayit 4:2, p. 235 in the Mossad haRav Kook edition) the principle of *sfek sfeka* may simply arise naturally from the iterative application of the rules for handling a single *safek*; the first *safek* reduces the issur to a *derabanan* and the second renders it permitted (see also *Shaarei Yosher* 1:19). A related interpretation, ascribed to R. Yosef Dov Soloveichik, is that we regard a *safek* between permitted and only *possibly* forbidden as if there is no *leidat hasafek*. Consequently, even in instances of *sfek sfeka* in which it might be possible to ascertain the actual facts, one is not obliged to make a special effort to do so (Rosh on Avodah Zarah, Perek Ein Ma'amidin, Para. 35 and Ramo YD 110:9). Understood this way, *sfek sfeka* has the effect of one type of *RDLK*: in each, there is no *leidat hasafek*.

A second approach to *sfek sfeka*, the one of primary interest for our purposes, extends the analogy to *rov* to include the mechanism through which *sfek sfeka* works. Thus R. Y. S. Natanzon (Resp. Shoel Umeishiv 1:196 and many other places) elaborates that *sfek sfeka* can be thought of as an extension of the idea of *RDIK* in which the sample space consists of the set of possible truth assignments to the various individual conditions[19]. For example, a newlywed woman who is found to have had sexual

[19] One disadvantage of this approach is that sometimes the phrase *sfek sfeka* is used with regard to a mixed set which then falls into another set (for example, Zevachim 74a). According to this second interpretation of *sfek sfeka*, the overlap in terminology is misleading and such mixture within mixture cases must be analyzed in the context of the rules governing mixed sets discussed above. According to the first approach, the conflation of ordinary kinds of *sfek sfeka* and mixtures within mixtures makes sense: in each the "outer" *safek*/mixture is treated leniently because the *chamur* side fails to satisfy the requirement of *itchazek issura* (see the discussion near footnote 12 above).

relations prior to marriage would be forbidden to her husband if she had done so (A) subsequent to betrothal and (B) consensually (Ketubot 9a). Of the four possibilities (subsequent/consensual, subsequent/not-consensual, prior/consensual, prior/not-consensual), only one renders her forbidden. Consequently, if we are in doubt as to which of these four possibilities is indeed the case, we can follow the majority and she would not be forbidden to her husband.

If indeed *sfek sfeka* is a type of *rov*, in its mechanics as well as its effect, it is certainly not *RDLK* since many examples of *sfek sfeka*, such as the one above, are not amenable to explanation in terms of *RDLK*[20]. Moreover, there are differences between *sfek sfeka* and *RDLK* even in terms of effect. We have already seen that R. Meir's principle of *chaishinan lemiuta* applies only to *RDLK* and not to *RDIK*. But in Nidda 59b we find that (according to R. Yohanan) R. Meir invokes a *sfek sfeka* against *chazakah d'me-ikara* and appears unconcerned with the principle of *chaishinan lemiuta*. This is consistent with *sfek sfeka* as an extension of *RDIK* but not as *RDLK*[21].

[20] For example, the question of whether a particular woman is more likely to have had relations subsequent to or prior to her betrothal would appear to depend on the circumstances of the case in question; there is no evidence of any general statistical rule that the Rabbis applied to all such cases. For more convincing examples of the inapplicability of *RDLK*, see M. Taharot 6:4 and Nidda 33b. It is important to note that there are cases of *sfek sfeka* and pseudo-*sfek sfeka* which happen to *also* be *RDLK*. For example, in Hullin 77b, Rashi s.v. *vechol hayoldot* (perhaps based on JT Yebamot 16:1) formulates the gemara's claim that most births are not both living and males in terms of *sfek sfeka* even though stillbirths are clearly a minority. The reason that this works is that it happens to also be a *RDLK* (See *Shev Shmaatse* 1:18). Similarly, Rambam *Hil. Para Aduma* 9:16 employs the terminology of *sfek sfeka* to explain the gemara's claim (Hullin 9b) that *mei chatat* that are found uncovered can be assumed not to have been handled by an agent who was both human and *tamei*. This would contradict the law that *sfek sfeka b'reshut hayachid tamei* except that it also happens to be a *RDLK*. See also Bava Kama 11a, Tosafot s.v. *d'eino*.

[21] Still, it must be noted that a majority of possibilities as in *sfek sfeka* is not always treated in exactly the same way as a majority of concrete objects as in the usual case of *RDIK*. In Tosefta Taharot 6:2 (as explained in Ketubot 15a), we find that in a case of *safek tumah* involving *parish* (the word used in the Tosefta is *nimtsa*), we follow the majority even for leniency; only in a case of *kavua*, do we follow the rule that *safek tumah b'reshut hayachid tamei*. Nevertheless, in M. Taharot 6:4 we find that *sfek sfeka* regarding *tumah be-reshut hayachid* is an inadequate basis for leniency. On the face of it, this would suggest that *sfek sfeka* is treated differently than *RDIK*. See Resp. Achiezer [Yoreh Deah] Resp. 2, Par. 13.
It would appear more appropriate, however, to ascribe this anomaly to the generally exceptional nature of the principle *safek tumah b'reshut hayachid tamei*. As is evident from the Tosefta, *safek tumah* refers specifically to cases of *kavua*, which, as we have seen, is an atypical use of the term *safek*. Thus, it would appear that the *sfek sfeka* cases in the Mishnah are regarded as more similar to the Tosefta's *kavua* case than to the *RDIK* case. In fact, Rav Soloveichik argues that the rule that *safek tumah b'reshut hayachid tamei* employed in the Mishnah applies only when the presence of *tumah* is certain so that the only uncertainty involves whether it was *metamei* other things. Thus all cases of *safek tumah* for which the rule is relevant may full under some broader definition of *kavua* (alluded to in footnote 16 above with regard to *asham taluy*). Consequently, the rule is inapplicable in the *nimtsa* case of the Tosefta in which *RDIK* is applied to a single object which might not be *tamei* at all. (It is difficult, however, to reconcile this restriction to the rule with the language of the Mishnah where the rule is applied: "*safek haytah sham, safek lo haytah sham*".) See *Shiurei HaRav Aharon Lichtenstein* on Taharot, pp. 167-171. (For another example of the atypicality of the rule *safek tumah b'reshut hayachid tamei*, see footnote 18 above.)

The most intriguing aspect of the identification of *sfek sfeka* with *RDIK* is the consequent link to the classical interpretation of probability. The issues that are raised by the commentators regarding restrictions to the application of *sfek sfeka* are remarkably similar to those subsequently raised with regard to the classical interpretation of probability. Consider, for example, the statement that "the probability that the sum of two throws of a die will be exactly six is 5/36". The classical interpretation of this statement is that in a sample space consisting of 36 possible throws of the dice, 5 have the desired condition. But, as we have seen earlier, this choice of sample space is not inevitable; it must be stipulated. Admittedly, this particular sample space is intuitively sensible based on two hard-to-pin-down "indifference principles":

- P1. For each toss, the six possibilities are plausibly symmetric.
- P2. The two tosses are plausibly independent – that is, the possibilities for each toss remain plausibly symmetric regardless of the result of the other toss.

But critics of the classical interpretation have pointed out that any attempt to regard the choice of a sample space based on such principles as anything more than mere stipulations inevitably runs aground on at least one of the following two problems:

- Q1. Primitive claims of plausible symmetry must not be confused with precise claims of equiprobability. To do so would be to invite infinite regress by introducing probability into the definition of probability.
- Q2. The same problem sometimes permits different plausible symmetries which may lead to contradictory conclusions[22].

Consider now the central issues regarding the applicability of *sfek sfeka*. Recall that the idea is that both A and B are necessary conditions and both are in doubt. Rishonim have limited the applicability of *sfek sfeka* to cases in which two "indifference principles" can be invoked:

[22] A number of interesting examples of this phenomenon, all beyond the scope of this paper, are subsumed under the name "Bertrand's Paradox". See von Mises (*op. cit.*), p. 77.

- P1'. A and not-A, and B and not-B, respectively, are plausibly symmetric in the sense that *rov* can't be applied to either one of them (Ketubot 9a Tosafot s.v. *v'ee ba-it*).
- P2'. A and B must be independent ("*sfek sfeka ha-mit'hapechet*")[23].

Thus, for example, if there were a firm rule that most cases of sexual relations are consensual, our newlyweds' marriage above could not be saved by *sfek sfeka*. Moreover, the *sfek sfeka* works only because the question of consent is regarded as independent of the question of timing (prior to or subsequent to betrothal), in the sense that resolving one question would not resolve the other.

In fact, Rishonim are at pains to point out that the plausible symmetries underlying the definition of *sfek sfeka* must be regarded as mere Rabbinic stipulations:

- Q1'. Rivash (Resp. 372) points out that symmetry in these cases should not be confused with equiprobability.
- Q2'. Tosafot notes that by formulating the problem differently we might find plausible symmetry between A&B and not-(A&B) thus destroying the *sfek sfeka* (e.g. "*shem oness chad hu*", Ketubot 9a Tosafot s.v. *v'ee ba-it*).

Continuing with our example, no claim is made that the probability that relations were consensual is precisely ½. It is enough that there are two possibilities, consensual or not-consensual, and no firm rule rendering one more likely. Moreover, as Tosafot notes, the possibility that the relations were non-consensual could in principle be counted as multiple possibilities by distinguishing violent rape from statutory rape. It is a matter of stipulation that we do not do so but rather bundle all cases of rape under a single label.

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[23] Halichot Olam, Agur and Rivash (Resp. 192) attribute this view to Tosafot but without citing a source. Indeed it is not found in the standard Tosafot but can be found in Tosafot Yeshanim, Ketubot 9b (cited in the outer margin in the Vilna shas) . The Shach in YD 110:9, Gloss 63 (*Dinei Sfek Sfeka*), Para. 13-15, points out that in many known cases of *sfek sfeka* the independence condition fails and that in fact we require only partial independence – namely, that where there is a natural ordering on A and B (say, A is logically prior to B), it is sufficient that the question of B or not-B not be resolved given that A but not vice versa. Thus, for example, we find (M. Taharot 6:4) a *sfek sfeka* in which the two conditions are (A) that one entered a room in which there is a *tamei* object and (B) that one touched the object. Obviously, if B holds then A holds as well. Still, since the question regarding A arises prior to that regarding B, this is a valid *sfek sfeka* (albeit an ineffective one for unrelated reasons). In this case, there are not four possibilities, as in the standard *sfek sfeka*, but rather three (the possibility that one touched the object but was not in the room is eliminated). Still, there is a majority of two out of three.

In short, attempts to explicate *sfek sfeka*, like attempts to explicate the classical interpretation of probability, contend with the need for some fuzzy symmetry criterion, more primitive than equiprobability, on the basis of which to choose a sample space[24]. This parallelism makes it tempting to see in *sfek sfeka* precisely the kind of extension of *RDIK* required to complete the analogy with the classical interpretation of probability.

Since very few cases of *sfek sfeka* are actually considered in the gemara, it is unclear how broadly we might apply it. In the most restrictive interpretation, *sfek sfeka* would only apply where two conditions are required and each condition permits precisely two possibilities: yes or no. All the cases in the gemara are of this type[25]. In a broader interpretation, the breakdown to independent conditions might just be a shortcut for counting elementary cases to determine if a majority of them permit or forbid. For example, suppose widgets came in three colors, {red, white, blue}, and three sizes, {small, medium, large} and that a widget is forbidden precisely if it is (red OR blue) AND (medium OR large). Then, by the broader interpretation, a particular widget of unknown color and size might be permitted since only 4 of 9 possibilities are forbidden. By a narrower interpretation, uncertainty regarding color and uncertainty regarding size would each be resolved independently in favor of a 2 of 3 majority to forbid and the widget would be forbidden.

According to the broader interpretation, constraint P1' above should be interpreted as strictly analogous to P1: for purposes of case counting, the cases must be plausibly symmetric but there need not necessarily be precisely two possibilities within each constituent *safek*. Note that even according to this broad interpretation, *sfek sfeka* does not provide a blanket license for multiplying numerical probabilities. It merely offers a shortcut for counting cases. As explained at great length by the Shach (YD 110:9, *Dinei Sfek Sfeka*), where case counting is inapplicable, such as when the constituent *sfekot* involve conceptually incommensurable aspects of the situation, *sfek sfeka* does not apply.

[24] See the discussion in Rabinovitch (*op. cit.*), Chapter 5.

[25] A number of arcane exceptions are considered by later commentators. A summary can be found in E. Cohen, "Introduction to *sfek sfeka*" (in Hebrew), *Sinai* 112 (5753), pp. 273-282.

Summary

To summarize, Rabbinic methods for resolving uncertainty regarding the status (P or not-P) of some object are non-numerical but rather yield one of the following outputs:

- certainly P / not-P
- probably P / not-P
- uncertain (*safek*)
- hybrid (*mechtsah al mechtsah*)

These methods can be roughly summarized by the following procedure:

1. If the object is known to belong to some fixed set containing objects that are P and others that are not-P, then use the rules outlined above to apply either *RDIK* or *kavua*. When *RDIK* is invoked, the conclusion is either probably P or probably not-P. When *kavua* is invoked, the conclusion is *mechtsah al mechtsah*.

2. If the object is not known to belong to some fixed set, but does belong to some class about which we have some recognized statistical law (*RDLK*) concerning the property P, then assign its status according to that law. Depending on the strength assigned to the law, this status might be regarded as certain or as merely probable.

3. When the necessary conditions for P or not-P to hold can, according to Rabbinic judgment, be most naturally formulated in terms of a multiplicity of plausibly independent symmetric *sfekot*, we resolve in accord with the majority of theoretically possible instances (*sfek sfeka*). This conclusion is regarded as probable.

4. If none of the above allow resolution, the status of the object is *safek*. In such cases, second-order default rules might be invoked to determine a course of action. These second-order rules involve the nature (*tumah*, *yuchsin*, *mamanot*, or *issurin*) and severity (*deoraita* or *derabanan*) of the prohibition in question and various presumptions (*chazakot*), a detailed discussion of which is beyond the scope of this article[26].

[26] See P. Schiffman, "*Safek* in halakhah and in law" (in Hebrew), *Shenaton haMishpat haIvri* 1 (5734), pp. 328-352.