JEWISH CALENDAR IN THE ROMAN PERIOD:

IN SEARCH OF A VIABLE CALENDAR SYSTEM

Ari Belenkiy

Mathematics Department, Bar-Ilan University, Israel

Introduction

The calendar system which Jews use now was known to Babylonians at least at the end of the 4th cent. BCE. This allows Jewish apologetes to claim that this was the ancient Jewish system copied by the Chaldeans after the conquest of Judea in the 6th century BCE. Even so, after the second national catastrophe in 70 CE, we see a remarkable break with this tradition - Jewish leaders rely on immediate observations of the new moon to fix the first day of the month and the ripeness of fruit to intercalate an additional month in the lunar calendar rather than mathematics. There are no signs that the astronomical achievements of ancient Greece and Babylon were used by Jews in the first five centuries of the Christian Era.

Meanwhile, defeats in the two great wars against Rome in 70 and 135 CE caused a flow of refugees to the neighboring countries, mainly to Babylonia. At first the notes about all the decisions rendered by the calendar council were passed by fire signals or by messengers, but soon both systems were inadequate. This caused Jews to look for a fixed calendar system. We discuss here two such systems. They are simple and can be called "arithmetical.c This period is fairly well recorded in the Talmud, albeit with significant omissions, and we have to reconstruct some missing parts of those systems. For this purpose the evidence of Christian authors, contemporaries of these events, are most important. In fact, this period was a Jewish-Christian competition for the best calendar system.

Time Reckoning by Jews in Antiquity

The most audacious Torah reference to calendar matters is the command "keep the spring month and keep the Passover unto the Lord" (Deuteronomy, 16: 1). When, coming from Babylonian exile in the 4th century BCE, Ezra introduced the lunar calendar to Judea, this Biblical verse was understood as "keep the lunar year in line with solar year."[1] Probably reflecting the practice of the Jewish priests in the 1st cent BCE - 1st cent. CE, Josephus in *Jewish Antiquities* (III, 10:5) and Philo the Alexandrian say[2] that the *"full moon of Passover always had to fall after the Vernal Equinox, when the sun stood in the sign of Aries."* Yet the details of the computations and the procedure of adding a 13th month in the time of the Second Temple are not historically documented.[3] After the national catastrophe - the destruction of the Second Temple in the year 70 CE - a special rabbinical court [Bet Din] consisting of 3-7 members[4] gathered annually to vote on the addition of an extra, 13th, month. The reasons to add or not to add a month were agricultural -- the ripeness of barley and the flourishing of fruit trees - and seasonal.[5]

Because the history of the Jewish calendar is enshrouded in mystery, the Jewish sages of the Middle Ages took differing attitudes toward the significance of the calendar. In the fierce polemics about the calendar which broke out in 922 CE in the Jewish community, Babylonian Jewish leader Saadiya Gaon of Baghdad claimed that "the calendar was given to Jews at Mount Sinai" and thus is sacred, unchangeable, eternal.[6]

Maimonides (Egypt, c. 1170) in the treatise "The Sanctification of the New Moon," of his fundamental code of law *Mishneh Torah*, took a more cautious stand. Not discussing the ontological significance of the calendar, he argued (ch. 5:1) that the use of a fixed calendar is a necessity of Diaspora life. When the Jewish Supreme Court, the Sanhedrin, will once again function in Jerusalem, the first day of the month will again be determined by the testimony of witnesses about the new moon and the additional month in the lunar year will be determined according to the well-known agricultural signs. Maimonides, however, agreed that Sanhedrin might use mathematical computations for facilitating its decisions.

When the Bet Din began using mathematics for intercalating years and when mathematics entirely supplanted agricultural signs is unclear. In 992 Hai Gaon, head of one of the two

Jewish academies in Babylonia, in one of his epistles credited the version of the calendar we use now to Hillel II, or Hillel bar Yehuda Nasi, a Jewish leader in Eretz Israel c. 359.[7] The reasons for making the calendar public were given as the difficulty of swiftly informing the Diaspora about the timings of holidays and a fear that the procedure would be forgotten and lost.[8]

The modern Jewish calendar has a 19 year intercalation cycle with intercalary years 3, 6, 8, 11, 14, 17, 19 (consisting of 13 months). The calendar month, a distance between two consecutive *moladot*, is accepted to be equal to the "mean lunar month" found by Ptolemy: 29 d. 12 h. 44 m. 3 1/3 s. To compute the time of the coming Rosh Hashana one has to find the *molad* of the previous Rosh Hashana and add 12 calendar months if the year is regular or 13 months if the year is intercalary. The result should be divided by 7. The remainder (after discarding minutes and seconds) will show the day of the coming Rosh Hashana. The minutes and seconds are also important - they imply four rules of postponing Rosh Hashana by 1-2 days. These four rules are, however, unrelated to the calendar core and are beyond the scope of this paper. [9]

Though the calendar is not known to have changed from Hai Gaon's time to ours, nothing substantiates his claim: there is much evidence that some details of the modern calendar appeared after the year 359. Nor are we sure what the calendar was prior to Hillel II, because Talmudic references to the calendar are very scarce and somewhat confused. As Nachmanides (Spain, c. 1250) wrote in his commentaries on Maimonides's *Sefer Ha-mitzvot*, our calendar is

"a matter of tradition and halakha [Jewish Law] from Moshe at Sinai, which is not mentioned in the Talmud or recalled in any other place."

The major evidence in support of Hai Gaon's claim is a certain Baraita in the Talmudic tractate Rosh Hashana ascribed to Rabban Gamliel of Yavne (d. 116), which quotes the exact old Babylonian (and also Ptolemaic) value of the mean lunar month and which also serves as the basis for the modern calendar system. The Baraita was the subject of long controversy in Jewish literature; critics proved that it suffered at least two augmentations and the last fragment was probably added in late 8th century Baghdad. Accepting this, the author suggests the timing and authorship of the first augmented fragment. It is connected to

the 3rd century Jewish astronomer Shmuel Yarkhinai (Babylonia, d. 254) and might represent the original Jewish contribution to ancient astronomy. The fact that the exact (or even close to exact) value of the mean month was unknown to Jewish leaders prior to the 3rd century leaves room for the possibility that other systems were previously in use.

The first such system is mentioned four times in Talmud and is known as the "theory of others." Though widely discussed, it was never properly explained in the literature. In the second chapter the author gives it a consistent explanation and show that this system was likely in use by the Jewish community after 359 CE. It was based on a simple arithmetic operation of annually shifting ("epact") the date of Easter 11 days down the Roman calendar, with 30 day jumps up at the intercalary years. The basic principle of this system is known in history: in the 1979 paper "Ethiopic Easter Computus" Otto Neugebauer described it as the calendar system of the Alexandrian Church of the 4th century. Having said that the latter borrowed its calendar from the Alexandrian Jewish community, Neugebauer did not supply any arguments to support the claim. Therefore identification of the *Epact* system with talmudic "theory of others" may fill in the gap.

Though there is no direct evidence that "theory of others" was used by the Jewish community, such evidence exists with respect to another, simplier system. One of the Church Fathers, Julius Africanus, reported (c. 212) that "Jews, like Greeks, add three months during an 8 year cycle." Though the existence of a system employing an 8 year cycle was never acknowledged in the Jewish tradition, the Talmud hints that somewhere between 170 and 220 CE there was such a calendar system. It was built on much a simpler principle of shifting each holiday's day of the week by four days and can be called the "week day" system. The latter is rediscovered by juxtaposing it with the phrasing of the "theory of others."

In the last chapter possible timing of the switches between these systems and the modern one were suggested. In the conclusion the implications of our findings were discussed, chief among them that the myth of the Sinaitic origin of the modern calendar system must be taken cautiously and that the system accepted by Hillel II in the year 359 was likely a 19 year variant of the "theory of others."

I. Baraita d'Rabban Gamliel

1. CALENDAR MONTH

The Baraita in Tractate *Rosh Hashana* 25a says:
"Once the sky was clouded and the image of the moon appeared on the 29th day. People decided that it should be the beginning of a new month and asked the Bet Din to confirm this. However, Rabban Gamliel said: According to tradition from my father's house the moon does not renew itself for less than 29 days and a half and two thirds of an hour and 73 parts." [10]

Rabban Gamliel (d. 116 CE) was president of the Supreme Jewish Court in Yavne after the destruction of the Second Temple. Though Rabban Gamliel was an older contemporary of Claudius Ptolemy of Alexandria, he passed away some time before *Almagest* (c. 150 CE), in which the Greek astronomer gave the above value as the length of the mean lunar month,[11] was published. However, that value has an earlier history. Ptolemy himself attributed its discovery to Hipparchus of Rhodos (c. 150 BCE), claiming that the latter deduced this value from a comparison of his own eclipse observations with observations made by Babylonians some 350 years before. However, German scholars of the 20th century, notably Franz Xavier Kugler and Otto Neugebauer,[12] have shown that this value had been the basis of Babylonian luni-solar calendar, in use at the beginning of the Seleucid era (c. 280 BCE).

In principle, Rabban Gamliel's family could have been acquainted with the work of Greek astronomers -- they could read Greek and were not afraid of Greek wisdom.[13] Besides, the great-grandfather of Rabban Gamliel, Hillel ha-Zaken (fl. c. 0), came from Babylon. And yet....

2. CRITIQUE

The first critics of that Baraita were the Jewish astronomers and philosophers of early 12th century Spain. First Abraham bar Chiyah Savasorda[14] noticed c. 1123 that in a medieval (c. 776) text known as *Baraita d'Shmuel* [15] there is no trace of the "73 parts" but only of "two thirds of an hour." In his commentary (c. 1150) on Exodus 12:2, his younger

contemporary, Abraham Ibn Ezra, repeated the claim. This meant that the Baraita d'Rabban Gamliel suffered an augmentation.

However, the golden age of Spanish Jewry with its critical thinking was over and students of Nachmanides' school suggested only apologetic works. The next critical comment came from Bohemia in the wake of Copernican discoveries. David Gans (d. 1613) was a student of Rabbi Moshe Isserles of Poland and of the Maharal of Prague, and a scholar in his own right who cooperated with Tycho Brahe in the Prague observatory. In his last treatise, dedicated to a description of the new (Copernican) astronomy, Gans[16] noticed that another medieval text, *Pirkei d'Rabbi Eliezer* (ch.7), also lacks "73 parts." Having also found that a third medieval text, known as *Yalkut*, has "793 parts" instead of "two thirds of an hour and 73 parts," he concluded that not only "73 parts" but also "two thirds of an hour" was a later augmentation to the Baraita d'Rabban Gamliel.

In the second part of the 19th century a scholar from Warsaw, Chayim Zelig Slonimsky, made an in-depth study of the problem. Slonimsky[17] pointed out that the words *"not...for less"* in the text of the Baraita eliminate the opportunity to discuss a *mean* lunar month. As for a *true* lunar month -- given what we know now, it can be as short as 29 days and 6 hours. Besides, the argument in Baraita was about "days," not "hours." His conclusion was that not only "73 parts" and "two thirds of an hour," but also "half of a day" were inserted later. Indeed, the only way to make sense out of the Baraita is to say that a "true month is not less than 29 days." This means that the first augmentation Baraita suffered could have been of a different type.

However, with a mean month of 29 1/2 days it is still impossible to build a viable calendar - its faults will be obvious in a matter of 1-2 years. Only a value which includes "two thirds of an hour" allows applying the Molad technique consistently over a prolonged period.

4. SECOND AUGMENTATION

In the beginning of the 20th century a scholar from Warsaw, Chayim Yehiel Bornstein, noticed[18] that the unit of time "part" was not known to the Tosefta, a rabbinical source parallel to the first part of the Talmud, the Mishna, written c. 200. The first paragraph of the

Tosefta *Berachot* speaks about different units of time incompatible with "part".[19] This implied that if the Baraita appeared in the 2nd century then it must have been without its last part "73 parts."

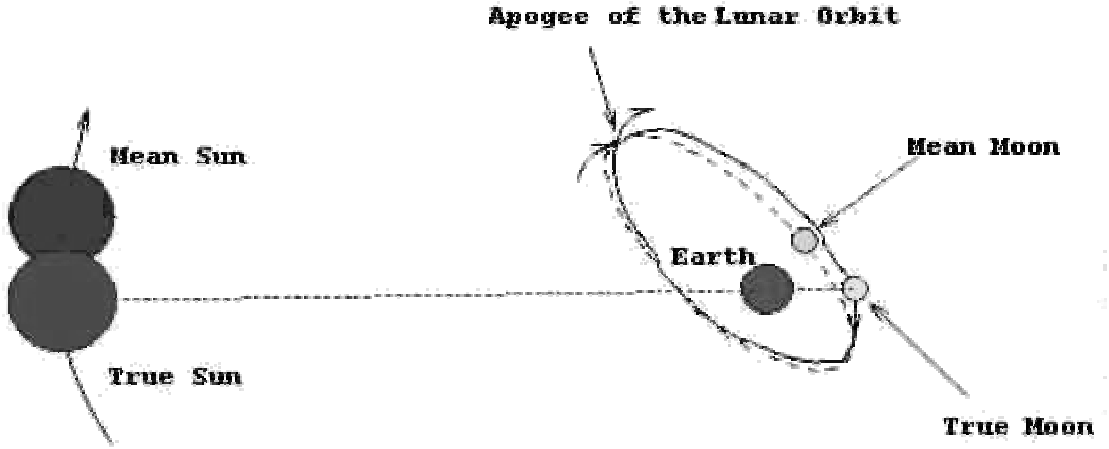
When could "73 parts" (equal to 4 minutes + 1 part) appear in the text of the Baraita? Though there is evidence that Ravina, the last editor of the Talmud (d. 499), was aware of the 4 minutes,[20] the fact still remains that the Baraita's basic unit of time --*"part"* -- was unknown at the time of the final editing of the Talmud, c. 500. Such a basic notion could appear only when new cultural circumstances demanded it. One such circumstance could be a renewed interest in astronomy, when the old books were republished and old theories checked anew.

The first such epoch after antiquity was in late 8th century Baghdad under the enlightened Caliph Hārūn al-Rashid, patron of astronomy. Though the most authoritative translation of *Almagest* in Arabic was completed c. 828 CE, there was a translation in Aramaic (Syriac) -- the language of Talmud -- half a century before.[21] Giving the year 776 CE as a calendar epoch, the last (likely added later) chapter of *Baraita d'Shmuel* might point precisely to that activity. At this point everyone could realize that the Jewish calendar did not have a unit of time capable of capturing the exact *Ptolemaic* value of the mean month, the "3 1/3 of a second." That is when the *part* was invented and "73 parts" was added to the Baraita in the Talmud to bring the Jewish month of 29 *1/2* d. 40 min. to the exact Ptolemaic value.

5. FIRST AUGMENTATION

In this paper we are concerned with the timing and authorship of the first augmentation: "half of the day and two thirds of an hour." Though it holds the value of the mean month as 4 minutes less than the exact value, it still can be used to maintain a calendar based on the "mean month" (Molad) technique. This value is unknown in the Greek and Babylonian traditions and is probably a Jewish contribution to ancient astronomy. *Baraita d'Shmuel* suggests a probable author and timing for this text, we suggest one more - astronomical - argument.[22]

Only one who performed his own observations of the moon could arrive at the value *291/2 d. and 40 minutes* for mean month -- with its noticeable error of 4 minutes compared to the exact value given in the Baraita: *291/2 d. 44m. 3 1/3 s.* The contemporary theory of the moon, anticipated by ancients, can explain the exact circumstances of those observations. The observed (real) moon and sun behave quite irregularly and at different points of their respective orbits have different speeds.[23] The value for the mean month was set by Ptolemy as the time between oppositions of the abstract *mean* moon and *mean* sun which (both) move uniformly in the sky. It is known that at *syzygies*[24] the mean moon is never farther than 5^{o} from the true moon and the mean sun is never further than 2^{o} from the true sun.[25] So the maximal cumulative angular distance between true and mean eclipses cannot exceed 7^{o} degrees.



LUNAR ECLIPSE. TRUE MOON is AHEAD of the MEAN MOON.
TRUE SUN is BEHIND the MEAN SUN.

Observing the central moment of a pair of lunar eclipses with time interval T and N months apart, one could arrive at the value **T/N** for the mean lunar month, which is close to exact value within a certain error. In our case, the error should be about 4 minutes.

With its daily angular distance increase from the sun of about 12.2^{o}, the moon covers 7^{o} between true and "mean" oppositions in about *14 hours*. The observational errors in fixing

the central moment of eclipses are negligible.[26] Therefore the value **T/N** could not be farther from the exact value of the mean lunar month than 28/N of an hour.

It follows that N=420 months between eclipses will bring the error mentioned above up to the required 4 minutes (*28 hours / 420 = 4 minutes)* and any greater number will only make the error smaller. An error of four minutes could only be possible if the two eclipses were no more than 420 months, or 34 years, apart. Such a short period implies that someone performed his own observations. The only possible observer in the Jewish world between late antiquity and the Muslim period was Shmuel Yarkhinai (d. 254), head of the Talmudic academy in Nahardea, near Babylon. Indeed, Shmuel was the accomplished Jewish astronomer who even boasted knowing *"all the ways of the moon in the sky"* and claimed to calculate the calendar tables 60 years in advance.[27] Remarkably the medieval text *Baraita d'Shmuel*, attributed to Shmuel Yarkhinai, also says that the month is *"29 1/2 days and two thirds of an hour."* [28]

From the NASA table[29] one can extract a long list of pairs of lunar eclipses for Shmuel's life span (fl. c. 200-254) which, after application of the formula above, gives a value close to 29.5 day and 40 minutes. For the vast majority of pairs the time interval between eclipses is no longer than 20 years and the maximal interval is 27 years apart.[30] The best match gives a pair of two total eclipses occurring on the 13th of December of 205 at 23:38 and on the 6th of March of 220 at 21:10, a time interval of 14.5 years, which perfectly matches our theory.

6. BARAITA'S AUTHORSHIP

The only way to claim that the original Baraita contained the words up to "two thirds of an hour" is to switch its authorship. It might belong to a Rabban Gamliel, but a different one -- not the Rabban Gamliel II of the 2nd century. A contemporary of Shmuel Yarkhinai was Rabban Gamliel, son of Rabbi Yehuda ha-Nasi. In this case his Baraita might be a public recognition of the value found by Shmuel. Why, then, could the words "not for less" appear in the Baraita? This also could make sense. Shmuel could have made a series of observations whose pairing gave a set of different results for the mean value of the month, and "29.5 days and 40 minutes" was a solid lower boundary for this set.

We conclude by restating that the value of the mean month which would allow the building of a calendar similar to the modern one was not known to the 2nd century Jewish community. This provides a chance to believe that prior to the modern system other calendar systems were tested.

II. "Theory of Others" and "Epact" Theory

1. CRUCIAL FACT

Proof that the fixed calendar was introduced no later than the second half of the 2nd century comes from a dispute in the Talmudic tractate *Rosh Hashana* 19b between an anonymous sage (Rabbi Nathan or Rabbi Meir?) and Rabban Simon ben Gamliel about the length of the additional month intercalated in the Jewish intercalary year -- the former suggested intercalating 30 days while the latter 29 days (a "month"). If it was not a fixed calendar, then as we mentioned above, the beginning (and the end!) of any month had to be decided by accepting the testimony of two witnesses.

2. "THEORY of OTHERS"

In four tractates and in five places the Talmud[31] mentions the following passage:

"And others say: from Atzeret to Atzeret and from Rosh Hashana to Rosh Hashana - 4 days only, though in an intercalary year - 5 days."

This statement is known as the "theory of others" and might be attributed to the "others" -- either to the dissident scholar Elisha ben Abuya (fl. c. 120-50), to Rabbi Nathan (fl. c. 150-90), or to Rabbi Meir (fl. 130-90). The first "cut the roots" -- stopped observing Jewish law -- and was excommunicated at the time of Rabban Gamliel. The second sage was a student of the first and was third in authority in the time of Rabban Simon ben Gamliel. The third sage came from Babylonia and was second in authority in the time of Rabban Simon ben Gamliel. The latter two suffered expulsion from the Talmudic Academy in Usha.[32]

Starting with the famous Rashi (1040-1105), a commentator on Torah and Talmud, the "theory of the others" was commonly understood[33] as meaning two things:
1) that the regular Jewish calendar year (of 12 lunar months) must contain 4 days *on top of* a whole number of weeks; the only reasonable number is 354 days.
2) the Jewish intercalary year (of 13 months) must contain 5 days *on top of* a whole number of weeks; the only reasonable conjecture leads to 383 days.

3. "INTERCALARY YEAR" in the "THEORY of OTHERS"

Logical as it sounds, the above explanation encounters serious difficulties. The 383 days in the intercalary year implies that the 13th month should be of 29 days. Then by simple arithmetic we get 6929 (354 * 19 + 29 * 7) days in the lunar cycle, while the solar part has 6939 3/4 days. The difference of almost 11 days is intolerable and the "theory of others" seems completely untenable unless we come forward with a different reading.

The only change which should be made is to reevaluate what the word "intercalary" means. In the recent paper "Sod Ha-ibbur: Shalosh Shitot B-luach Ha-ivri B-meot Ha-rishonot Le-sphira" this author suggested that "intercalary" means a **"Julian leap"** year and not a **"Jewish intercalary"** year.[34]

With this change the "theory of others" immediately becomes intelligible: it suggests adding a day to a Jewish year which overlaps with a Julian leap year thus allowing the regular Jewish years to consist not only of 354 days but also of 355 days. Let us show that this system fits well into a 19 year cycle (with its 7 intercalary and 12 regular years).

First note that without adding an additional day every Julian leap year, the calendar has on the lunar part only 6936 (354 * 19 + 7 * 30) days**,** whereas the Julian solar calendar has 6939 3/4 (365 1/4 * 19) days during 19 years.

To make the lunar calendar go along with the solar we have to change the number of days in the lunar part. In every 19 years there are on average 19/4 Julian leap years.[35] With these 4 3/4 days we can get, in the lunar cycle, 6940 3/4 (6936 + 4 3/4) days.

We see that the lunar 19 year cycle is one day greater than the solar cycle. Though the way to solve the problem is obvious (omit one day from the lunar calendar) the "theory of others" is silent on this. It is also silent on the name of the 13th month and the sequence of intercalary years.

4. "EPACT" SYSTEM

Remarkably, the "theory of others" is known in world history, though in disguise. In one of his last papers, "Ethiopic Easter Computus", Otto Neugebauer described the system used by the Alexandrian church in the beginning of the second quarter of the 4th century.[36] Though all the Ethiopic tables came from the 15th century, one of the documents contained the date 44th year of Diocletianus (=327 CE) and the table coincided with Pascal messages of the Alexandrian Patriarche Athanasius. The fragments of the calendar which survived also contain the dates of all major Jewish holidays and Neugebauer was convinced that this system had origins in the Alexandrian Jewish community. The system was called, in the Greek and medieval Latin literature, the *Epact*, where "epact" meant the difference between the solar and lunar years.

The civil calendar used in Alexandria in the Roman period was a mixture of the old Egyptian calendar and the Julian calendar. In the latter all the months are of 30 and 31 days and the additional 366th day was February 29, but in the former all months were of 30 days and 5-6 extra days were added only at the end of the year, in August. So the beginning of the Alexandrian civil year (*Toth* 1) fell usually on August 29, except in the years preceding Julian leap years, when an extra day was added on that day, so that year (*Toth* 1) started on August 30.[37]

The Alexandrian church calendar was extremely simple and consistent. All Jewish holidays each year were moved, with regard to the Alexandrian civil calendar, 11 days down and in the intercalary year additionally moved 30 days up.[38] The system of epacts during the 19 year cycle was: ***0, 11, 22, 3, 14, 25, 6, 17, 28, 9, 20, 1, 12, 23, 4, 15, 26, 7, 18,*** so an epact at year **i** was computed by the formula $\mathbf{e_i = (e_{i-1} + 11) \bmod 30}$. The civil date of Rosh Hashana in the year **i** of the cycle was then computed as $\mathbf{30 - e_i}$*:*

30, 19, 8, 27, 16, 5, 24, 13, 2, 11, 10, 29, 18, 7, 26, 15, 4, 23, 12,

where the dates smaller than 14 stand for the second month *Phaophi* and the dates greater than 14 belong to the first month *Toth*.

We see that continuing to subtract 11 days, the next cycle should start not on the 30th of *Toth* but on the 1st of *Phaophi*.[39] This was prevented by moving all the holidays down by 12 days in the last year of the cycle. This 12-day shift in the medieval literature was called "saltus lunae" - "jump of the moon."

5. Equivalence of the "theory of others" and "epact"

The *Epact* system is equivalent to the "theory of others." Indeed, an 11 day shift down with respect to the Alexandrian civil calendar gives a 354 day lunar calendar year, but the same shift done in the Alexandrian civil leap year leads to the lunar calendar year of 355 days. This is exactly what the "theory of others" suggests.

Because Rosh Hashana in the *Epact* system fell not earlier than in the mid-September and thus always later than the Alexandrian 366th extra day, the only way for the Alexandrian Jewish community to keep 11 day jumps consistently for all Jewish holidays from Rosh Hashana to Passover was to add the 355th day to the lunar calendar before Rosh Hashana, in the preceding month of Elul (which regularly contained 29 days).

The Talmud tells us about [at least] four cases in which the month Elul was made 30 days long and those emendations were done before the year 359.[40] The difficulty is that according to the Talmud this happened quite rarely, whereas according to the "theory of others" it should be done at least every 4 years (with a possible break due to the "saltus lunae" described below.)

The "theory of others" is seemingly silent on when to add the 355th day. But, in fact, detailed study reveals the true scope of its succinctness. Of course, it is concerned with the preservation of the 11-day jumps system, but there is more in it. Saying "from Atzeret to Atzeret" before "from Rosh Hashana to Rosh Hashana" means that adding the day should be performed in the interval the two periods overlap: from Rosh Hashana to the following Atzeret (Shavuot). And because between Passover and Shavuot there are exactly 50 days, it

implies that the intercalation of a day has to be performed during the first six months following Tishrei, in one of the 29-day long months, Heshvan, Tevet, or Adar. Such a move will spoil the 11-day jump system in the Alexandrian civil calendar. However, it points to another system which has the 366th extra day during that period - the Julian calendar. This means that the "theory of others" was oriented to the major Roman calendar system and thus was truly independent of Alexandrian church calendar and most likely goes back to the Jewish academy in Usha at the end of the 2nd century, where the "others" taught. In the preference of Nisan over Tishrei (or Passover over Rosh Hashana) one can also discern genuine Jewish tradition of counting years from the Spring which only later (after the closure of Talmud) was supplanted by the *Tishrei* tradition.

In the calendar of the Alexandrian church, as well, we see that the distance between Rosh Hashana and the "Passover feast" (Nisan 14) was always of 190 days,[41] so the intercalary month in the lunar calendar was Elul II. The system of intercalations of the second Elul goes back to the ancient Babylonian calendar [42]

6. 30-year-cycle "epact" system

Because "others" are silent about "saltus lunae," let us look for another *Epact* system which does not need it. The only such system is a 30-year cycle. Subtracting 11 days each year with addition of 30 days in the intercalary years, after 30 years (with 11 years intercalary) one comes to the same starting date in the Julian calendar.

The 30 year cycle is also known in history: it was championed by the Eastern (Antiochean) Church, as is seen from the *Sardica Document*, submitted by group of Eastern bishops to the Sardica Council in 343.[43] It is remarkable that this group tried to prove that Jews might use exactly the same, 30 year, cycle: in that document the dates of Christian full moons are parallel to Jewish Passovers for the last 16 years; these dates coincide except the cases where Passovers fell before the Equinox. Unfortunately, one cannot conclude from 16 years which cycle Jews actually used: a 30 year cycle or a 19 year cycle with "saltus lunae."

7. "Saltus Lunae" in the 19 year cycle "epact" system

If Jews used the *Epact* system with 19 year cycle in practice, one has to explain what they did with "saltus lunae," because there is no place for the year of 353 days in a calendar with Elul the only variable month. The only solution is that the day was dropped not in the last, 19th, year of the cycle, but in any Julian leap year of the cycle: for example, the 30th day in Elul was simply not added. Then one can understand the following otherwise obscure passage from Talmud (*Rosh Hashana* 21a) about the practices of Rava (d. 350), a head of the Talmudic academy in Sura (Persia):

"Rava would observe two days of Yom Kipur [in case Elul was a full month], and he was once found justified."

This could only mean that Rava knew in advance when Elul had to be 30 days and that in years when Elul was supposed to be 30 days he still celebrated Yom Kippur two days - on Tishrei 9 and 10 (by his reckoning). As we know, Yom Kippur falls on Tishrei 10 and Tishrei 9 was not supposed to be celebrated. So how was Rava "once found justified"? Because Persian Jewry later learned that in that year the Calendar Council in Eretz Israel did not extend Elul to 30 days! Then Yom Kippur had to fall on Tishrei 9 by their reckoning - so Rava "was found justified."

The suggested procedure of not allowing one year to be 355 days long (12-day jump down the calendar in the leap year instead of the standard 11-day jump) makes the use of the standard "epact" table less convenient and, as we believe, forced the next emendations of the calendar. The resolution of the new problems led to the rising of the modern *Molad* system.

8. HISTORICAL PRIORITY

It is important to find what was first: Church or Jewish community, and who borrowed from the other: the "theory of others" or the Alexandrian church calendar. There is an interesting episode in the Palestinian Talmud (*Erubin* 21c):

"Rabbi Abahu went to Alexandria and they raised lulavin on Shabbat. Rabbi Ami heard and said: who will bring them Rabbi Abahu every year? Rabbi Yosi sent them a message: even

though we wrote to you the dates of the festivals, keep the customs of your fathers, whose souls are at rest."[44]

Rabbi Ami (a disciple of Rabbi Yohanan) and Rabbi Abahu lived at the end of the 3rd century. Rabbi Yosi probably lived at the same time or slightly later. Whether the dates sent to Alexandria were computed from the "theory of others" remains an open problem which we will deal with in the subsequent papers. If proved, the passage above would confirm Otto Neugebauer's belief that the Jewish community in Alexandria has priority before the Church in use of the "epact" theory.

9. What was the "theory of others"?

Was it 19 or 30 year cycle system? Taking the Talmudic statement at face value it is the latter. Another place where 30 years emerge in connection with "others" is tractate *Arachin* 9b where Ravina, objecting to "others," said:

"but isn't it one extra day every 3 years and one day in 30 years?"

Bornstein took it to mean adding 11 days on top of the regular 354 day year pattern (with no intercalation, as Muslims still do), which leads to the mean month equal to 29.5 days and 792 parts, only one part less than the true value. According to him, Ravina's 30 years have nothing to do with a 30 year intercalation cycle, but have to be taken as a rebuke to the "others'" value of a mean month. Though both cycles are off the rebuke with greater weight can be addressed to the 30 year cycle (with mean month by 6 min. greater than the true one) than to the 19 year cycle (with mean month by only 20 sec. greater). Because this pattern is purely lunar with no intercalation of the 13th month, Bornstein assumed that the last part of Ravina's statement ("one day in 30 years") was added to the Talmudic text much later, in the Muslim period.[45]

We suggest, however, another reading of the passage. Ravina's 30 years might have something to do with a 30 year intercalation cycle. It can hardly be an interchangeable pattern of 29-30 days,[46] because the addition of an extra day would lead to three 30 day months in row. Ravina could mean that intercalatory month, added 11 times over 30 years, was always of 30 days, thus arguing against "saltus lunae" and a 19 year cycle.

We suggest that different Talmudic schools in Tveria, Nehardea (Pumbedita), and Sura might have favored different cycles. The possibility of "saltus lunae" accidentally slipping away from the statement of the "others" in five places in the Talmudic text is negligible. Such a coordinated effort, however, could be easily achieved by an editor, Rav Ashi (c. 425). In this case we can think of the 30 year cycle favored by the Babylonians against the 19 year cycle of Eretz Israel.

III. The Eight Year Cycle

1. HISTORICAL EVIDENCE

A famous Christian author of the 4th century, Eusebius, quotes in his *Demonstrations Evangelique* (VIII, 2, 54) his predecessor Julius Africanus, one of the first Church fathers, who wrote in his "Chronicle" that

"Jews, like the Greeks, intercalate three months every eight [lunar] years." [47]

Julius Africanus lived in Eretz Israel at the beginning of the 3rd century (which is the end of the Mishnaic period) and had a thorough knowledge of local customs. Besides, he is considered the father of Christian chronology. His five books of "Chronicle" (Gk. *Chronographia*) cover the time from Creation (B.C. 5499 by his calculation) to the third year of Eliogobalus (221 CE). Heinrich Gelzer thinks he wrote this work between the years 212 and 221 CE. [48]

These years mark the end of reign of Rabbi Yehuda ha-Nasi (d. 220), the "father" of Mishna. Two facts are relevant in this context. First, during this period Samaritans made trouble for Jews in transmitting the day of the new month to the Diaspora in Babylonia (Parthia). The Talmud Bavli (*Rosh Hashana* 22b) said that Samaritans began lighting false fires on the top of their mountains on the 30th day of the month to mislead Jews in Syria and Babylonia. The best way for Jews to counteract this practice was to create a fixed calendar. Another fact is related to the clause "like Greeks" in the quote above. According to the Talmud Bavli (*Pesachin* 94b) Rabbi Yehuda was an open-minded leader who was not afraid of the wisdom of "other nations," so he might have copied the system used by the population in the surrounding Greek cities (like Tiberias or Ptolemaus).[49]

2. "WEEK DAY SHIFT" SYSTEM

What was the actual system employed by Rabban Simon ben Gamliel and his son Rabbi Yehuda? The Talmud is silent on this. We can recover it by juxtaposition with the "theory of others" described above. By juxtaposition with the "theory of others" and choosing "Occam's razor" one may conclude that it was ruled by the principle *"from Rosh Hashana to Rosh Hashana - 4 days,"* or that each regular Jewish year of 12 months always has to be 354 days. Practically, this means that the next year holidays were four days later in the week. We call this system a "week day shift" system.[50]

The additional month intercalated in the Jewish intercalary year, of course, has to be 30 days, as we learned from the abovementioned dispute in Talmud' between an anonymous sage and Rabban Simon ben Gamliel. The fact that the opinion about 30 days was anonymous indicates that it was incorporated as a law.

The "week day shift" system perfectly matches an 8 year cycle (with its three intercalary and five regular years): there are 2922 (354 * 8 + 30 * 3) days in the lunar cycle, and the same number of days 2922 (365 1/4 * 8) days in the solar cycle.[51]

2922 days in an 8 year cycle with its 99 months lead to a mean month of *29 d. 12 h. 21.5m.* which is about 22 minutes shorter than the correct one. Therefore the calendar moon in the 8 year cycle runs 1.53 days faster than the real moon. After application of one or two such cycles this would quickly become a practical problem. The "week day shift" system had to be corrected. Remarkably, there exists an almost perfect cure.

3. "CORRECTED 8 YEAR CYCLE"

To save 8 year intercalary cycle one has to add 1.5 days per cycle or 3 days per two cycles. With this addition the 8 year cycle is only 0.03 days smaller than the astronomical one[52] and such a cycle can endure for a very long time; a one day discrepancy between true and calendar moons will accumulate in about 266 years. With 1.5 days added we will call such a cycle a "corrected 8 year cycle".

There is, however, no unique algorithm to add these 1.5 days inside 8 year cycle. One can suggest an *Epact* system with one "saltus lunae" in every two cycles. The "saltus lunae" can be implemented as addition of 29 day intercalary month once in two cycles. A hint in favor of such algorithm comes from Talmud (*Rosh Hashana* 19b) where a certain sage said that Rabbi Simlai, c.250 CE, testified that concerning a pair of Adar I and Adar II Bet Din could make them both full (30+30), both defective (29+29), or one full and another defective (30+29). The third option is regular. The first option implies *Epact* system with an extra 355th day in Adar II. The second option might reflect the "saltus lunae" in Adar I as explained above.

The same sage also said that "*according to our rabbi"* (Rabbi Yehuda ha-Nasi?) the first option was forbidden. Thought it is unclear whether "our rabbi" lived earlier or later than Rabbi Simlai, this could reflect a switch not of an entire system but only in intercalation pattern. The absence of a 30+30 pattern in the 8-year cycle could mean that the intercalation sequence was not allowed to intersect with a Roman leap year, being for example, 2-5-7 or 1-3-6.

Of course, choosing such a system, the Jews had to look for guidelines. The fact that a "corrected" 8 year cycle can endure for more than hundred years should be either checked empirically, which takes significant time, or confirmed by contemporary astronomy. Of course, educated Jews could read *Almagest* and find a 1.5-day correction, which closely matches Ptolemaic value for a mean month. Besides, in the 2-3rd centuries CE there could be other astronomical treatises, written by Ptolemy's contemporaries, like Theon of Smyrna, Menelaus, or by any other astronomer quoted by Ptolemy or not quoted and now forgotten, who might give slightly different values for mean month,[53] so Jewish sages were not necessarily alarmed by imperfect matching with Ptolemy's mean month. However, Jewish sages might have had their own empirical data.

4. WERE JEWISH SAGES AWARE OF MEAN MONTH?

The Jewish sages might have had a notion of the "mean month" and might have discussed its estimates even being unaware of astronomical works. At the time immediately following the destruction of the Temple in 70 CE, Jewish procedure for fixing the first day of the new

month was to interrogate two witnesses. For this purpose the Calendar Council used to gather on the 30th day of every month. If two witnesses appeared before the Council on that day, they were interrogated; if their testimony was accepted that day was announced as the first day of the new month -- leaving the previous month as 29 days long. If two witnesses fail to appear or their testimony was discarded as unreliable, the next day was announced as the first day of the new month -- leaving the current month with 30 days.

The Calendar Council likely had statistics on how many times witnesses appeared on the 30th day, and if this happened in less than half the cases one could conclude that the average month is greater than 29 1/2 days. If for example, such statistics was held for 8 years and of 99 cases testimony was accepted in 48, then they could conclude that the mean month is about *29 d. 12 h. 40 m.* For a relatively quiet period between the two revolts in the years 70-130 CE such statistics could have been even more impressive and led to a better estimate.

One can argue about the consistency of such statistical estimates. For example, witnesses expected a reward if their testimony was accepted, so they were willing to come. There is also evidence that Rabban Gamliel accepted very spurious testimony, which makes it possible that he had a certain preconceived idea of how many months in a row should consist of 29 or 30 days.[54] Even so, these cases would not spoil overall statistics: they could increase variance in the sample and give a bad approximation over a short period but not over a prolonged period.

IV. Switches Between Systems

1. WHAT CAN INFLUENCE A SWITCH OF CYCLE?

The reason for a switch from the 8 year intercalation cycle to the 19 year cycle could be obvious: the former, when corrected, in the end of the cycle slips off the Julian calendar whereas the latter gives a perfect match. However, the 8 year intercalation cycle has a more serious fault. After some 150 years in use it would require intercalation of an extra month compared to the 19 year cycle. This would lead to a jump off the equinoxes toward the summer. This can be seen from the following simple arithmetic. For 304 years a system based on the Julian calendar and a 19 year cycle requires 111036 days, whereas based on an

8 year cycle 111093 days.[55] The extra 57 days roughly amount to 2 extra intercalated months.

The advantage of a 19 year cycle can be seen from following procedure. Though the year may consist only of 12 or 13 months (but not, say, of 11 or 14 months) it is not immediately clear that the set of permissible cycles is limited. However, the procedure for finding an appropriate cycle has been well known since the golden age of Greek mathematics. Assuming a lunar year being equal to 12 months, we have to expand the expression

$$F = \text{solar year / month}$$

in the continued fraction[56]

$$F = 12 + 1/(a+1/(b+1/(c+...))))$$

and look for its first rational approximants:

$$1/a,\ 1/(a+1),\ 1/(a+1/b),\ 1/(a+1/(b+1)),\ 1/(a+1/(b+1/c)),\ 1/(a+1/(b+1/(c+1/d+...)))).$$

The denominators will show us the permissible cycles and numerator -- the number of intercalary years in each cycle .

Different values for month and year can dramatically change the set of the permissible cycles. For a *month* of 29*1/2* days and a Julian year of 365 1/4 days we have

$$F = 365\ 1/4 \,/\, 29\ 1/2 = 1461 / 118 = 12\ 1/(2+1/(1+1/(1+1/(1+1/(1+1/(1+1/(5...)))))))).$$

The first rational approximants are *1/2, 1/3, 2/5, 3/8, 5/13, 8/21, 29/76, 37/97*... We see the absence of 19 and 30 year cycles. The fraction 3/8 is present; therefore the 8 year cycle can be applied.

However, for a month of 29*1/2 d. 40 min.* and a Julian year of *365 1/4 days* this fraction is

$$F= 365\ 1/4 \,/\, 29\ 19/36 = 13149 / 1063 =12\ 1/(2+1/(1+1/(2+1/(2+1/(1+1/(2+1/(12...)))))))).$$

The first rational approximants are *1/2, 1/3, 3/8, 7/19, 10/27, 17/46*... and the 19 year cycle immediately follows the 8 year cycle as a better (more precise) candidate. Notice that the 30 year cycle is entirely absent for these values of mean month.

2. SWITCH TO the 19 YEAR CYCLE

When did the 8 year cycle give way to the 19 year cycle? Shmuel left for Babylonia c. 200 and disciples of Rabbi Yehuda ha-Nasi, like Rabbi Yohanan who came in his place after his death in the year 220, were unwilling to make any changes to the calendar. Talmud Bavli (Chulin 95b) says about disparaging comments by Rabbi Yohanan about Shmuel: *"he knows only the calendar!"* Rabbi Yohanan died c. 250, so the switch, if it did occur, was after 250 CE.

We can only speculate that the most appropriate time for it was a new era -- Era Diocletianus which started in September of 284. As we know, Jews never shunned using the popular time reckoning of the people amongst whom they lived, as for example the Seleucid era. However, recently Brendan McKay and the author found that the 19 year cycle was not in use up to the end of the 3rd century, though a 8 year cycle could have been used.[57]

The transition to 19 year cycle might have occurred in the first quarter of the 4th century. In a 327 CE letter by Emperor Constantinus, quoted by Eusebius in his *Vita Constantinus* (III, 18-20),[58] the emperor warned against any association of Easter with the Jewish Passover on the grounds that the Jewish calendar was in complete disarray and different communities celebrated Passover a month apart. Though the faults of Jewish computus were not explained, we can suggest that c. 327 CE there was a genuine disagreement in the Jewish communities of Eretz Israel and Alexandria about the intercalation cycle.

3. SWITCH TO the MOLAD SYSTEM

So what happened in CE 359? Because of the obvious arithmetic**: 284 + 19 * 4 = 360,** the most probable case is that in CE 359 Hillel II announced the introduction of a new system starting the next year, CE 360. This year initiated another era, acquiring the name "Era of Mercy." However which system did he use: the "theory of others" or the Molad system?

The traditional version suggests the second, *Molad* option. To justify this claim we have to accept at least two things. First, that mid-4th century Jewish sages became aware of the Ptolemaic system in general and the Ptolemaic value for the mean month in particular. This could happen due to, e.g., the activities of Theon of Alexandria, the author of - or

commentator on - *Handy Tables.*[59] Such awareness could lead, in principle, to the Molad system. Second, that 3 1/3^{s} were consciously omitted from the Ptolemaic value due to the absence of an appropriate unit of time and the inconvenience of computing fractions with large denominators. However, there are no indications whatsoever of Theon's involvement in the Jewish matters.

Therefore the first option looks more palatable. First the "theory of others" involves only simple arithmetic. Besides, it is remarkably precise - enough to stay on the historical scene for a long period. It fixes 6939 d. 18 h. for a 19 year cycle, only 1.5 hours more than the *Molad* system, which for 235 months gives 6939 d. 16.5 h. (29 d. 12 h. 44 m.3 1/3 s. * 235). With that, a one day difference between the two accumulates over a period of 306 years. So if it was accepted c. 359, then only in the 8th century would the difference become conspicuous (more than a day difference between true and calendar moons) and could cause serious problems to the Jewish community.

4. Baraita d'Shmuel

We have indirect proof that in 776 CE the "theory of others" was still in use. The 5th chapter of the *Baraita d'Shmuel* (added later to the main text of the manuscript) indicates that in the year 4536 Anno Mundi (776 CE) *"the sun and moon became equal."*[60]

This could mean that Molad Tishrei and Tekufa Tishrei (Autumnal Equinox) fell on the same day. The answer to the question of whether there was then a capable astronomer among Jews can be answered positively: Māshāllāh, a builder of Baghdad.[61] A simple check, however, shows (see Appendix) that Molad Tishrei of 776 CE was on September 17, 21:20[62] while the Autumnal Equinox in 776 CE fell on September 19, 11:30,[63] so the difference was more than 1.5 days. This invalidates the conjecture - it is impossible to miscompute Equinox by two days. The statement above must mean something else.[64]

We suggest that the phrase meant that the day of Rosh Hashana (Tishrei 1) fell on the same day as the Autumnal Equinox. The "theory of others," as confirmed by the Ethiopic manuscripts quoted by Neugebauer, does satisfy this requirement. First, 416 years passed since 360 CE, suggesting that at the end of the 8th century CE the calendar moon of the

"theory of others" was later by some 1.3 days than the true "mean moon." An examination of the Ethiopic tables shows that in the 18th year of the cycle (in particular, 776 CE) Tishrei 1 had to fall on September 20.

There are two different explanations of how Molad Tishrei on September 20 and the Autumnal Equinox on September 19 "were the same." The first is that Jews did not find the Autumnal Equinox from the observations but computed it from the book by Ptolemy, who, as is known, miscomputed the Equinox by 28 hours.[65]

Another explanation is related to the "saltus lunae." If it was inserted in the calendar, as we suggested above, not in the 19th year of the cycle but in the last Alexandrian leap year of the cycle (by canceling an extra day in Elul), then it was done in the year 775 CE. Therefore in the year 776 CE Molad Tishrei had to fall on September 19 - a day earlier compared with the Ethiopic tables.

However, the major argument in favor of Hillel's adoption of the "theory of others" is epistemological: if the editor of the Talmud, Rav Ashi, left only data relevant to his time (425 CE), then the system promulgated by Hillel II in 359 CE was the 30 year or 19 year "epact theory" cycle with intercalation of the 13th month Adar I instead of Elul II. That "saltus lunae" is missing in the Talmud speaks in favor of a 30 year cycle; taking Hai Gaon's statement literally leads to a 19 year cycle.

From the 5th century CE until the mid-8th century (when Māshāllāh arose) there is only one leader in the Jewish community capable of making the next switch - Elazar ha-Kallir, whose period of life is uncertain and dated to the 6-7 centuries. There are indications that he used a 19 year cycle. If it was he introducing that cycle then "saltus lunae" demanded the second variable month and Heshvan and Kislev stepped into the place of Elul. The following Baraita might be associated with him.

5. MEDIEVAL BARAITA

Though there is no direct proof that Jews ever used the 19 year cycle "theory of others," indirect proof can be found in the *Baraita* quoted by medieval Jewish writers.[66] This

Baraita brings a dispute between the Jewish leaders Rabbi Elazar and Rabban Gamliel. They argued about the sequence of intercalary years in the 19 year cycle:

Said Rabbi Elazar: three, two, three, three, three, two, three.
Sages said: three, three, two, three, three, two, three.
Rabban Gamliel said: three, three, two, three, three, three, two.

In other words, Rabbi Elazar wanted to intercalate years 3, 5, 8, 11, 14, 16, 19 while the sages years 3, 6, 8, 11, 14, 16, 19 and Rabban Gamliel years 3, 6, 8, 11, 14, 17, 19.

The Baraita likely discussed intercalation of the month Adar I, whereas in the Alexandrian church calendar Elul II was added in the 2, 5, 7, 10, 13, 16, 18th years of the 19 year cycle. If the Baraita also assumes Era Diocletian as an epoch, the civil dates of Passover suggested by Rabban Gamliel and in the church calendar would be absolutely the same during the entire cycle and, moreover, reveal the nature of the disagreement among Jewish sages. According to Neugebauer, Passover of the Alexandrian church fell on (italics show the dates belonging to the 7th month of Alexandrian calendar *Phamenoth*, the rest - to the 8th month *Pharmouthi*): 10, *29*, 18, 7, *26*, 15, 4, 23, 12, 1, 20, 9, *28*, 17, 6, *25*, 14, 3, 22.

Looking at these dates we see that the above dispute among Jewish leaders makes perfect sense: The sages were concerned with the 16th year, where Passover falls on *Phamenoth* 25, and Rabbi Elazar -- not only with 16th year but also with 5th year, where Passover may fall on *Phamenoth* 26.

Notice that *Phamenoth* 25th always coincided with March 21[67] of the Roman calendar, which was the day Alexandrians believed the Vernal Equinox fell in the 4th century. So the question under debate was whether Passover can fall on the day of the Vernal Equinox. Zvi Ioffe argued that Jewish sages of the 2nd century had already discussed this question and claimed that Passover was allowed to fall a day before the Vernal Equinox.[68] This is inconsistent with the Ethiopic tables and with opinion of the Jewish leader Rabban Gamliel.

However, the sequence of intercalary years did not exactly follow the Church pattern! This can be seen from the 387 CE letter of Ambrose, bishop of Milan, where he wrote that *"Jews were going to celebrate Passover (feast?) on March 20."* Other Christian writers confirm

the early date of the 387 Passover.[69] On the other hand, the year 387 CE is the 8th year in the Alexandrian Church calendar, which, according to Ethiopic tables and Ambrose's opinion, had to be intercalated and the Church's "full moon of Passover feast" fell on April 18.[70]

This speaks either of a different starting point for counting the 19 year cycle or of a later dating for the Baraita quoted by Isaac Israeli. The year 360 is the 16th year in the cycle of the system we use now - counting the 1st year of the 1st cycle from the year -3760 (3761 BCE), which preceded the year of Creation by a year. It means that Jews of the 4th century did not count years from the Year of Creation and the notion itself might not have existed at that time.

EPILOGUE

Anyway, in 360 or in 776 CE the "theory of others" gave way to the *Molad* system used in the modern Jewish calendar. This means that the Jewish calendar passed through three different phases in its development*: 8 year cycle -> Epact system -> Molad.* All three are principally different. The first system, the "corrected 8 year cycle," is purely empirical, whereas the second system was tied to the Roman civil calendar. Only the last, modern system dropped the goal "to conform" altogether, concentrating exclusively on the lunar part of the calendar, leaving its solar part to pay the price of the natural vicissitudes of a 19 year luni-solar cycle.

The circumstances behind the introduction of an 8 year cycle were not unique to the period of 170-220 CE - Jews experienced the same difficulties between the two great revolts of 66-70 and 132-35 CE. At that earlier time Jewish leaders were people whom Jewish tradition venerates as the most brilliant people of Talmudic period: Rabbi Akiba and his friends. Because there are some implicit hints in Talmud about Rabbi Akiba's use of the 8 year cycle, one can conclude that this cycle was used by the Jewish community in the time of emergency and that it became usable at least 100 years earlier than Julius Africanus' remark.

These two additional systems destroy any basis for Saadia Gaon's claim that the "Jewish calendar originates from Mount Sinai." Otherwise one is puzzled why Jewish leaders in the era after the destruction used other systems. However, this conclusion awards us much more

-- it brings back the calendar's history. It also changes the perception of Jewish intellectual history between the 2nd and the 10th century, which now can be seen not solely as a chain of Talmudic studies but also as a continuous search for the most viable calendar. This search often was not straightforward and -- as numerous instances witness -- was rich with internal conflicts and excommunications.

Another conclusion is that the Talmud hides much more than it displays and that the final redaction by Rav Ashi resulted in the elimination of large pieces of historical data (like the use of an 8 year cycle). As we will show in subsequent papers, after the year 359 the Jewish calendar followed a long and painful[71] path of adjustments, introducing different *dekiyot* (rules for postponing the new year), fixing the *Molad*s (the points in time for counting the calendar moons), and finally choosing the first *Molad* -- the year of Creation. This process was slowed down by Saadia Gaon's intervention[72] and was over by the time of Hai Gaon.

Acknowledgements

My friend Alon Wulkan (of Givat Shmuel) elucidated Rashi's commentaries on the Baraita in tractate Sanhedrin. Benyomin Elboim (Bar-Ilan University) discussed possible schemes for an 8-year cycle as used by Rabbi Akiba. Althea Katz (of Har Bracha, near Mount Gerizim) helped with editing and references. Brendan McKay (Australian National University), upon my request, analyzed NASA tables and found a list of pairs of lunar eclipses which lead to the value of 29.5 days and 40 minutes. Daniel Michelson (Weitzmann Institute), with his letter, spurred my interest in the "theory of others," though his own explanation suggested too many changes to the Talmudic statement and thus is unsatisfying. I am grateful to the participants of the mini-conference "Link between Judaism and Christianity: Calendars and Time-Reckoning" organized by Prof. Dr. Joseph Wohlmuth and the Catholic Faculty of the University of Bonn in November 1999, in particular to Ariel Cohen (Hebrew University of Jerusalem) for emphasizing Saadia Gaon's role in birth of the myth about Sinaitic origin of modern calendar, Reinhold Bien (Astronomy Research Institute of Heidelberg) for bringing to my attention the important paper by O. Neugebauer, and to Heiner Lichtenberg (Bonn) for pointing to the use of Epact in the pre-Gregorian church's lunar calendar.

Appendix. Gauss's Formula for the Molad

"Gauss is always right" (Mathematical parlance)

Gauss's formula for Molad Tishrei:[73]

$$M + m = 183.0955877 + 1.5542418 *a + 0.25 *b - 0.003177794 *B,$$

where

M is the number of days between March 1 inclusively and Molad Tishrei,

m is a fraction of the day counted from noon of the previous day,

B is a C.E. year,

$a=\{\frac{12B+12}{19}\}$ is the is the remainder after division of (12 B + 12) by 19;

$b=\{\frac{B}{4}\}$ is the remainder after division of B by 4.

---

[1] Talmud, *Rosh Hashana* 21a. All references to Babylonian Talmud (Bavli) can be found in the Soncino edition (London 1938).

[2] Cf. Emil Schurer, *Geschichte des Judischen Volkes in Zeitalter Jesu Christi*. (1903-1924); English translation: *The History of the Jewish People in the Age of Jesus Christ* (Edinburgh: T&T Clark LTD, 1973), vol. 1, Appendix 3: 593.

[3] It is unclear which Aries was meant: the real constellation or the zodiacal one? The real physical Aries stretches over only 21^{o} in the sky (see *Almagest*, ch. VII.5, constellation 22:Aries), so Josephus likely meant the latter case. We could assume that Jewish priests were aware that Hipparchus of Rhodos (fl. c. 150 BCE) had identified the vernal equinox with the 1st point of Aries. So "when the sun stood in the sign of Aries" probably means during first 30 days after the vernal equinox. In the most economical 19 year cycle Passover stretches over 28 days. The equinoxes move on the sky 1^{c} per 72 years. So if observations by the Jewish priests were exact up to half-degree they have, say, a 150 year long pattern through which to identify the 19 year cycle and reject 8 year or 11 year cycles.

[4] Talmud, *Sanhedrin* 10b. The council started with three arbiters and in the case of disagreement and split of opinions invited two more people and then two more; seven was a maximal number.

[5] Talmud, *Sanhedrin* 11b. Barley had to be checked at least 45 days before Passover for the procedure known as Omer (cutting a sheaf on the eve of Passover as a symbol of starting count 50 days toward Shavuot). Fruit had to be ready for Shavuot (first fruit to the Temple) and had to be checked 95 days before that. The inquiry on "seasons" (or whether Passover was earlier or later than the Vernal Equinox) was done with a similar check to see whether the middle of the first winter month Tevet was before or earlier than the Winter Solstice.

[6] See *Kitāb Attāmyiz* (*The Book of Discernment*), where he defended the rabbinical calendar against attacks of Karaites. Saadia Gaon wrote in Arabic but his statement was often quoted by the next generations of scholars. See, e.g., Rabbeinu Chananel's commentary on Exodus, XII, in any version of the Jewish Bible in Hebrew.

[7] This epistle was quoted by Abraham bar Chiyah Savasorda of Spain in his 1123CE *Sefer Haibbur* (London: Filipowsky, 1851):97 and the date was written as the *"670th year of Selecuid Era."*

[8] The second reason is usually supported by the claim that it was a time of persecutions of the Jews by Emperor Constantinus II. However, it was when Julian the Apostate, friendly toward the Jews, ascended to the Roman throne. Besides, the next year, 360 CE, is known in history as the first year of the Era of Mercy.

[9] We discussed them in a separate study, see A. Belenkiy "A Unique Feature of the Jewish Calendar Dekhiyot," *Culture & Cosmos*, vol. 6, no 1 (2002): 3-22.

[10] 1 part = 1/ 1080 hour = 1/18 minute = 3 1/3 seconds; hence 73 parts = 4 minutes + 1 part.

[11] *Almagest*, IV: 2, gave it in sexagesimal system as $29;31,50,8,20^{d}$.

[12] F.X. Kugler, *Babylonischen Mondrechnung* (Freiburg: 1900): 7, 24, 111; see also discussion in O. Neugebauer, *The Astronomy of Maimonides and Its Sources*, HUCA, xxii (1949): 326.

[13] See Talmud, *Bava Kama* 83a, also *Sotah* 49a and its discussion of why Rabban Gamliel's family had to know Greek.

[14] *Sefer Haibbur, ibid:* 36.

[15] This treatise was published in 1861 in Salonici. It was widely discussed recently, see references in E. Beller, "Ancient Jewish Mathematical Astronomy," *Archive for History of Exact Sciences* 38, no. 1 (1988).
[16] D. Gans, *Nehmad ve-naim* (Yessnitz: 1743): 63.
[17] Ch. Z. Slonimsky, *Yesod ha-ibbur* (Warsaw: 1888):34.
[18] Ch. Y. Bornstein, *Makhloket Rav Saadyah Gaon u Ben Meir* (Warsaw: Dfus Shuldeberg ve-Shutafo, 1904): 115.
[19] The first is 1/24 of the hour, the second is 1/24 of the first, the third is 1/24 of the second.
[20] In tractate *Arachin* 9b Ravina says that to the regular sequence of interchangeable 29 and 30 day months 1 day should be added every 3 years (this refers to "2/3 of an hour") and, additionally, 1 day every 30 years (this implies 4 minutes). Slonimsky, *Yesod ha-ibbur*: 28, and Bornstein, *Makhloket Rav Saadyah Gaon u Ben Meir*: 116, considered the last words to be an addition made in the Muslim period. See also E.Weisenberg, *Code of Maimonides. Appendix*. Yale Judaica Series. v. 11 (New Haven: Yale University Press, 1961): 579.
[21] According to Toomer [*Ptolemy's Almagest*, ed. G. J. Toomer (Princeton: Princeton Univ. Press, 1998): 2], translations of the *Almagest* made in Islamic times were: 1. a Syriac version (8^{th} cen.) 2. a version made for Caliph al-Ma'mūn by al-Hasan ibn Quraysh (early 9^{th} cen.). 3. another version for al-Ma'mūn by al-Hajjāj and Sarjun ibn Hiliya (c. 828).
[22] A. Belenkiy, "Sod Ha-ibbur: Shalosh Shitot B-luach Ha-ivri B-meot Ha-rishonot Le-sphira," *Proceedings of the 11th Conference on the History of Judea and Samaria*. Ariel (2002): 275-86.
[23] This fact was known to ancients and, in particular to Ptolemy, see Almagest [III:3].
[24] New or full moons, including eclipses.
[25] See R. Newton, *Crime of Claudius Ptolemy* (Baltimore and London: John Hopkins Univ. Press, 1978) equation VI:7 regarding the moon and ch. VI:3 regarding the sun. Ptolemy's value [Almagest, IV:6] for the angular radius of epicycle (c. 5^{o}) leads to the close estimate for the moon, however he seemingly [Almagest, III:9] gives the estimate for the sun twice greater than modern.
[26] Ancient astronomers made errors no greater than 10-15 minutes, see *Crime of Claudius Ptolemy*, ibid, [VI:6].
[27] See Talmud, *Erubin* 56b and *Chulin* 95b. In 224 CE Sasanian Persia overthrew the Parthian kingdom and took their possessions in Mesopotamia. 60 years is a period of alignments for Jupiter, Saturn and the sun, which played a central role in the horoscopes of the royal Sasanian house. See, e.g., D. Pingree, *Historical Horoscopes*, JAOS 82 (1962): 229-46.
[28] It was shown that *Baraita d'Shmuel* contains elements of the 2nd-3rd century astronomy and its astrology coincides with that of Shmuel as recorded in Talmud, see "Ancient Jewish Mathematical Astronomy": 59-60.
[29] F. Espenac, "List of Eclipses for the Years 2000 BCE to 3000 CE." (NASA publication, 1998). Web site: http://sunearth.gsfc.nasa.gov/eclipse/Lecat/Lecatalog.html.
[30] Brendan McKay, personal communication. Time of the eclipses is UT time; one has to add 3 hours to find local (Baghdad) time.
[31] See Talmud, *Rosh Hashana* 6b and 20a, *Sukka* 54b, *Shabbat* 87b, *Arachin* 9b. "Atzeret" here means "Shavuot" celebrated 50 days after Passover.
[32] See Talmud, *Horayot* 13b. The reason for expulsion seems to have been unrelated to this topic.
[33] Cf. *Makhloket Rav Saadyah Gaon u Ben Meir*: 31, and most recently S. Stern, *Calendar and Community* (Oxford: Oxford Univ. Press, 2001): 159-60.
[34] *Op.cit*.: ch. II. In Talmudic times there was only one word, "*meuberet,"* for both leap and intercalary years. Only in later, Gaonic, times there was coined a special word for the Roman *leap* year.
[35] Of course, every particular 19 year cycle contains either 4 or 5 leap years. However, four cycles, i.e. 76 years, contain exactly 19 leap years.
[36] O. Neugebauer, "Ethiopic Easter Computus." *Oriens Christianus*, 63, no. 4 (1979).
[37] See, e.g., E.G. Richards, *Mapping Time: The Calendar and its History* (Oxford: Oxford University Press, 1998): 157.
[38] After finding the date of Passover and its day of the week, finding the date of Easter is an easy task.
[39] 7 intercalary months give 210 days, whereas 19 11-day jumps only 209 days.
[40] See Talmud Bavli, *Rosh Hashana* 20a,b and 21a,b. Though these four cases occurred over the course of four generations (the first -- during the life of Shmuel Yarkhinai, the last -- during the life of Hillel II), there might be other cases as well.
[41] The 190 stands for 177 + 13. The 177 days before the 1st of Nisan show that previous six months were of 30 and 29 days interchangeably.
[42] See *Babylonischen Mondrechnung*: 210.
[43] First analyzed by E. Schwartz, cf. *Calendar and Community*: 124-132.
[44] Cf. *Calendar and Community*: 173, though Stern's discussion of this passage is irrelevant here.

[45] *Makhloket Rav Saadyah Gaon u Ben Meir*: 130; cf. *Calendar and Community*: 203.

[46] Cf. *Calendar and Community*: 202-3. Following this logic, Ravina added 11 days on the top of the 29-30 day sequence and criticized the "others" for adding 30/4 "Roman leap days" and 11/2 extra days in 30-day-intercalatory months - overall 13 days. This means that Ravina's 30 year cycle was 2 days shorter than that of the "others." True, such logic can lead to an interesting conjecture. Because there are an odd number of months in Ravina's cycle, the number of days in it depends on the first month (29 or 30 days). So his actual cycle must be a 60 year one. This 60 year cycle was claimed by Shmuel Yarchinai in a letter to Rabbi Yohanan c. 250 (tractate *Erubin* 56a). It is interesting that Ravina's argument in *Arachin* 9b followed the claim that Shmuel allowed the year to be 352 and 356 days. If Shmuel was an author of Ravina's cycle then his problem was to find a way to add an extra 11 days every 30 years.

[47] Cf. *The History of the Jewish People in the Age of Jesus Christ*
"Greeks" in Julius Africanus's statement were "Syro-Macedonians." It could rather mean Alexandrian Greeks - at the same time the 12th Alexandrian patriarch Demetrius suggested his own computus, see Neugebauer, "Ethiopic Easter Computus": 526.

[48] H. Gelzer, *Sextus Julius Africanus und die Byzanntische Chronographie* (Leipzig: 1898): 6. See also Julius Africanus at the website http://www.newadvent.org/cathen/08565a.htm.

[49] Stern in *Calendar and Community*: 23, suggested absolutely implausible objections to the existence of an eight year cycle.

[50] Cf. A. Belenkiy, "Sod Ha-ibbur: Shalosh Shitot B-luach Ha-ivri B-meot Ha-rishonot Le-sphira": *ch. III.*

[51] Though 2922 days make a so-called "Venus cycle," when the sun, Venus, and Earth reach the same alignment with respect to the fixed stars, there is no evidence from the Talmud about Jewish sages of the 2nd century practicing planetary astrology. *Sefer Yetzira (The Book of Creation),* which G. Sarton in *Introduction to the History of Science* (Baltimore: 1927, vol. I, ch. III: 10) dates to the 2nd century, relates the 7 days of Creation to the 7 planets, not placing any particular emphasis on Venus.

[52] Which is equal to 99 times Ptolemaic month.

[53] If they compared own observations with eclipses observed by Hipparchus then they could not make mistake greater than half a minute. See our discussion above, in the 1st chapter.

[54] Talmud, *Rosh Hashana* 25a, where witnesses claimed to see a new moon that night and an old moon the night before. This case caused a scandal in the academy. To those who believed that Rabban Gamliel sincerely accepted the witnesses, Maimonides remarked on that case in his *Commentary on Mishna (Rosh Hashana, ch. 2:9)*: *"And no one believes this except an absolute fool and a boor who only perceives the world as the ox and the donkey would perceive it."*

[55] Interesting to notice that a standard 8 year cycle within "week day shift" system requires exactly 111036 days because two extra months will compensate for 1.5 day imprecision of the cycle itself. Of course, one should sacrifice common sense to wait for this occurrence for such a long period.

[56] The name was introduced by Rafael Bombelli in his *Algebra*, printed in 1572. Extracting the integers *a,b,c,d* ,...is also called the Euclidean algorithm for greatest common divisor of two integers. Note that it is the only way to find the right cycle. The simplicity of the procedure renders it available to any learned person since early times.

[57] See our forthcoming paper "Talmudic Puzzle and the Jewish Calendar in the Late Third Century."

[58] See also *Medieval Sourcebook: Constantine I: On the Keeping Easter* at
http://www.fordham.edu/halsall/source/const1-easter.html.

[59] This work is usually dated by c. 400, so the span of Theon's life was probably 320-400. His earliest astronomical activity we know about was his observation of the solar eclipse on June 16, 364 CE.

[60] See, e.g., "Ancient Jewish Mathematical Astronomy": 55. Though we counted Anno Mundi from Molad Baharad (-3760) while around 776 CE the Year of Creation was certainly counted differently than in the present-day and Molad Baharad simply did not exist, in the different copies of the manuscript there are indications of corrections and therefore the date in the manuscript we quote could be set much later, say, in the 10th cent. The major justification for accepting the year 776, however, is the fact that there is no other good candidate to compete with year 776 for Molad Tishrei being close to the A.E.

[61] Though Māshāllāh (d. c. 810) was a convert out of Judaism to Islam, he might have kept contact with Jewish community.

[62] See Appendix with B=776, a=14, b=0 and M+m=202.3890048 ≡ 9h. 20m. 10s. after Sept. 17, 12:00.

[63] Can be found by calculating back from the data supplied by al-Battānī, *Opus Astronomicum*. (Transl. into Latin and commentary by C.A. Nallino. Georg Olms Verlag: Hildesheim-N.Y., 1977), Part I: 212.

[64] A. Akavia in "Baraita d'Shmuel K-teuda Le-toldot Ha-ibbur" (Jerusalem: Melila, 1955) suggested that the "mean equinox" was actually meant in the Baraita. True, the mean equinox fell in 776 on September 17, c. 15:30 UT. The use of the mean equinox would, however, suggest that the Ptolemaic eccentric circle was

known to the author of the 5th chapter of the Baraita (this is plausible because there was an Aramaic translation of *Almagest* in the end of the 8th cent.) and that he was able to compute solar anomaly. To our knowledge the latter was computed for the first time after *Almagest* only c. 830 in Baghdad.

[65] According to R. Newton, *Crime of Claudius Ptolemy* [table 5.3], the vernal equinox in the year 140 CE fell on March 21, 9:30 AT, though Ptolemy fixed it as 28 hours later, on March 22.

[66] Isaac Israeli who flourished c. 1340 in Spain, says in *Yesod Olam* (Berlin: B. Goldberg and L. Rosencranz Publishing, 1846) Part 2, Article IV, ch.2:3a, that he "does not know original place of the dispute." Solomon Gandz in *Studies in Hebrew Astronomy and Mathematics* (N.Y.: Ktav Publishing House, 1970) associated the Baraita with a medieval text *Pirkei d'Rabbi Eliezer*.

[67] Be aware that *Universal Calendar Calculator* (http://www.cf-software.com/ucc.htm) gives the wrong date.

[68] Zvi Ioffe, *Qorot Cheshbon Ha-ibbur* (Jerusalem: Drom, 1931): 35.

[69] See, e.g., *Calendar and Community*: 144-5. Though John Chrysostom (*ibid*, ftn) quoted another date for Passover - March 19 - this confirms the fact that the Jewish year in 387 was not intercalated.

[70] This led to Easter falling on April 25 according to Roman computus and therefore to fierce polemics within the Church (*ibid*.).

[71] The sect of Karaites, who split c. 770 from main bulk of Judaism, pointed to imperfection of the rabbinical calendar as one of the pretexts for their split; see H. Malter, *Saadia Gaon: His Life and Works* (Philadelphia: The Jewish Publ. Soc. Amer. 1921): ch.4.

[72] After the famous struggle c. 922 against Aaron ben Meir, who tried to update the Jewish calendar according to al-Battānī's tables.

[73] See Gauss's original exposition in Gauss, C.F. *Berechnung Des Juedischen Osterfestes. Monatliche Correspondenz zur Beforderung der Erd- und Himmels-Kunde* (herausgegeben vom Freiherrn von Zach, Mai 1802) or its elaboration in Sh.B. Burnaby, *Elements of the Jewish and Muhammadan Calendars* (London, 1901): 219.