The Astronomy of Maimonides and its Sources

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Abstract. In this paper we analyze the fact discovered by Otto Neugebauer (HUCA, 22, 1949) explaining why the mean astronomical luni-solar conjunction set by Maimonides in his Code of Jewish Law “Mishneh Torah” differs from the *molad* (Jewish calendar conjunction) by 1 hour and 17 minutes. We also address Neugebauer’s fundamental question of whether molad is a mean conjunction.

This problem leads us to a further examination of Maimonides’ sources on the one hand and to a clarification of the notion of molad on the other. First we conjecture that a difference of *circa* 50 minutes between the epochs of Maimonides and al-Battânî came from a geographical manuscript which was different from al-Battânî’s treatise, translated by Carlo Nallino, which quoted an unreasonably high longitude for Raqqa (the location of al-Battânî’s observations) compared to Jerusalem’s longitude. Second, examination of a time difference between al-Battânî’s mean luni-solar conjunction and molad brings it close to 27 minutes. This shows that the interval of 1 hour and 17 minutes between Maimonides’ conjunction and the molad consists of only two parts and does not require the additional assumptions made by previous researchers.

"But of the things in the heavens Man knows nothing except a few mathematical calculations"
(Maimonides, "Guide for the Perplexed", book 2, ch. 24).

Background

As is well known, Maimonides (1135-1204) was an extremely versatile scholar. He was a major codifier of the Jewish law, and his *Mishneh Torah* (The Second Torah) remains one of the major legal sources for the Jewish orthodox community. The Code comprises 14 books, subdivided into 93 treatises.

The book on *Seasons* includes a treatise of 19 chapters, called *Sanctification of the New Moon,*[1] dealing with the calendar and astronomy. The major enigma surrounding these chapters is that they were written in two stages. The first ten chapters, written in 1170, basically repeat the simple astronomy of the Talmud with a solar year of 365¼ days [VI:4] and four equal seasons [IX:2].

However, Maimonides' study of astronomy with a disciple of Abû Bakr ibn al-Sâ'igh (called Ibn Bâjja) helped him to understand the principles of the Ptolemaic system and appreciate the innovations introduced by Muslim astronomers during the four centuries prior. In 1178 Maimonides added nine additional chapters to the initial ten. There he explained to the Jewish reader how to deal with Ptolemaic models in practice - using the parameters for solar and lunar motions found by Muslim astronomers. Very few Jewish writers dared to examine his computations.[2]

The first critical reviews of the *Sanctification* were written by German scholars Eduard Baneth in 1898-1903 and Otto Neugebauer in 1949. Baneth claimed to have discovered that among medieval scholars, Maimonides possessed the closest approximation to the value of the tropical year.[3] However Neugebauer,[4] after examining Maimonides' tables, concluded that he could have arrived at this result by pure accident, copying and rounding up mean velocities of the sun and moon from the manuscript of the Arab astronomer, Muhammad al-Battânî (fl. 882-940).[5] Besides, Neugebauer made an even more dramatic discovery.

Neugebauer's Discovery

"When dealing with the astronomical phase of the Hebrew calendar, Jewish scholars are inclined to be rather timid and faint-hearted. As a rule, the Jewish scholar has been trained as a philologist, historian and theologian, and it is therefore only natural that he should suffer from a feeling of inferiority when he is confronted with a positive and peremptory statement on the part of the reputable astronomer that the molad has nothing to do with the conjunction."

This is how Solomon Gandz[6] reacted to Otto Neugebauer's discovery that according to Maimonides [XII:3] the mean luni-solar conjunction had to be 1 hour and 17 minutes earlier than Molad Nisan, and to Neugebauer's conclusion[7] that the *"conventional statement that the molad [calendar conjunction] is the mean [astronomical] conjunction is proved to be false."*

Let us briefly review the topic, closely following Neugebauer's 1949 paper "The Astronomy of Maimonides and its Sources." In *Sanctification of the New Moon*, Maimonides [XI:16] fixed Nisan 3, $(5)^d\ 0^h$ (18:00), of the year 1178 as a base point (epoch) for computation of the sun's and moon's mean motions. We learn from XII:2 that at the epoch, the mean sun was at 7;3,32°, and from XIV:4 that the mean moon was at 31;14,43°. Thus the mean elongation at the epoch was 24;11,11°. We know from XIV:2 that the mean daily velocity of the moon is 13;10,35° and from XII:1 that the mean daily velocity of the sun is 0;59,8°. Thus the daily motion in elongation was taken to be 12;11,27°. Neugebauer noticed that from a comparison of luni-solar elongation at Maimonides' epoch and daily motion in elongation it follows that Maimonides' mean conjunction fell about two days earlier, on Nisan 1, $(3)^d\ 0^h\ 415^p$.[8] On the other hand, Molad Nisan 1178 fell on Nisan 1, $(3)^d\ 1^h\ 721^p$.[9] The difference is $1^h\ 306^p$, or exactly $1^h\ 17^m$. Neugebauer himself was puzzled:

"This would not be too interesting in itself if Maimonides did not seem to have overlooked the consequences of the definitions which he himself gave. In [VI:1] he seems indeed to indicate that molad means mean conjunction. The question arises how such a statement can be explained."

To defend Maimonides' integrity, Neugebauer offers a strange explanation:

"…it seems to me the way to answer can be found from the fact that Maimonides makes an equally wrong statement about Tekufot [equinoxes and solstices]…"

and further explains that the two halves of Maimonides' work differ: in the first (chapters I-X) Maimonides gives inexact but "simple" definitions whereas in the second (chapters XI-XIX, written eight years later) he deals more professionally with astronomical matters. Therefore

"Maimonides did not see any need to underline the fact that the astronomically accurate definitions would lead to slightly different moments - a fact which must have been evident to every astronomer of his time."

Apologists on the Jewish side did not clarify the matter. A philological attachment to the word “molad” prevented Gandz from understanding how to analyze the fact discovered by Neugebauer. His reply was highly inadequate and showed a poor understanding of the notion of *molad* as well as of *mean* conjunction. Other writers also have failed to treat this problem satisfactorily.

For example, in his pointed criticism of Neugebauer’s discovery, Ernest Wiesenberg[10] followed in Neugebauer’s footsteps but was only able to “reduce” the overall *77 minutes* problem to *31-35 minutes* by assuming that Maimonides could shift the standard Jewish 18:00 epoch up 42-46 minutes, including 10+4 minutes for the 18:10 apparent solar time of sunset in the late March = 18:14 mean solar time, plus an additional 20 minutes for new crescent visibility after sunset [XIV;6] and an extra 8-12 minutes for the “western extremity of the region for which Maimonides made his astronomical calculations.”

True, this explanation fits the framework of the classical Jewish apologetics - there is no hint that Maimonides could possibly have used external sources. However, neither of Wiesenberg’s arguments can be embraced wholeheartedly. They do not pass the “Occam’s razor” test. Nothing is known of Maimonides’ own observations of the sky or of his awareness of the notion of “mean time.”[11] The last 8-12 minutes allude to regions with a longitude 2-3 degrees west of Jerusalem, where allegedly Jewish witnesses could observe a new crescent not visible in Jerusalem on the night of the 30th day of the month. However, each degree of longitude amounts to about 56 miles and one cannot expect a man to cover such a long way in a single night to bear witness in the Bet Din before noon the next day.[12]

However, the major fault of the above apology is that it does not explain the artifact of 1^h17^m. Moreover, it obscures a simple fact discovered by Neugebauer, which is that the above interval is composed of two basic ingredients: geographical and astronomical. The main conclusion of our paper is that only two components are needed to account for 1^h17^m. The first is the difference between the epochs of Maimonides and al-Battânî due to a difference in the points of observation, the second is the difference between al-Battânî's mean conjunction and the *molad*. We address these consecutively in the first and the second parts of the paper.

I. LIBRARY OF MAIMONIDES

1. The "Unaccounted-for 23 Minutes"

Analyzing numerous tables in the second part of the *Sanctification of the New Moon* containing the mean velocities which Maimonides assumed for the sun and moon, Otto Neugebauer convincingly proved that these values were copied exactly, or rounded up from, the manuscript of Muhammad al-Battânî. However, comparing the positions Maimonides assigned to the mean sun and the mean moon at his epoch (beginning of the 5th night, Nisan 3, 1178) and those positions assigned by al-Battânî at his epoch (noon, Adhar1, 1178), Neugebauer found a discrepancy of 22 days, 6 hours, and 50 minutes.[13] Neugebauer correctly explained the 22 days and 6 hours as the time interval between epochs.[14] Then he attempted to explain the remaining 50 minutes but succeeded only partially. In Neugebauer's opinion, <u>27 minutes of the 50</u> were the difference in time between Raqqa and Jerusalem: al-Battânî gave 73;15° for longitude of the former and 66;30° for the latter.[15] The other 23 minutes remained unaccounted for. Neugebauer suggested that

"this amount corresponds most likely to the one third of an hour which Maimonides assumes (XIV:6) to be required after sunset in order to make the new crescent visible…Epoch was chosen in such a way that a delay of 20 minutes is already included. All that remains is a correction for variable length of daylight." [16]

Because the length of daylight is different every day, Neugebauer proceeds to compute it from al-Battânî's table of daily ascensions and then averages it over the year.[17] However, even after lengthy computations with multiple interpolations Neugebauer did not succeed in converting "one third of an hour" into 23 minutes.

Besides, this solution is purely hypothetical - Maimonides' treatise doesn't show any trace of such computations. Neither do we know about Maimonides' observations of the sky and his awareness of the practical problems involved. It would be more convincing to find 50 minutes in a single piece coming from one source of data.

Because Maimonides' parameters of lunar and solar motions came from al-Battânî, Neugebauer took it for granted that Maimonides used al-Battânî's geographical tables as well. However, we question this assumption. Arabic medieval sources are so numerous and give such a wide spectrum of longitudes for Raqqa that Maimonides, working three centuries after al-Battânî, could choose another geographical tractate for guidance assuming (in principle correctly) that a later source would be more reliable than an earlier one.

Before starting the search for such a tractate, let us make a digression pointing out to the two problems related to Muslim geographical books. The first problem is related to two different modes of counting - the first mode (coming from Ptolemy) from the Fortunate (Canary) isles (which was also al-Battânî's choice); the second mode - from the western point of Africa (al-Khwarizmi's choice). The two modes differ by exactly 10 degrees in longitude. Though irritating for a beginner, this is not a very dramatic problem since the difference between any two cities remains the same, and after seeing several entries, a user can easily understand which mode of two was actually used by the writer.

The second problem is more difficult to handle. The numbers in Arabic medieval treatises were represented by letters (with or without dots). Those representing numbers "3" (gimel) and "8" (het) are very similar in Arabic, differing by a single dot below, which, moreover, was often omitted or missed by scribes. The same can be said about the numbers "10" (yud) and "50" (nun). (See details in the introduction to the volume *Geographical Coordinates of Localities from Islamic Sources* composed by E.S. and M.H. Kennedy.[18]) Because the coordinate for Raqqa's longitude contains a "3" it could have been mistaken for an "8". Such a mistake would lengthen the distance between Raqqa and Jerusalem by 5 degrees, or 20 minutes, and would bring al-Battânî's 27 minutes close to the 50 minutes we are looking for. This suggests a working <u>hypothesis</u>: either the source that Maimonides used had the digit "8" instead of “3” in Raqqa’s longitude, or Maimonides misinterpreted one for the other by himself.

This fact that such an error factually occurred is confirmed by the existence of a geographical treatise listed by the Kennedys under the name *GT1*, whose text contains 78;15° for Raqqa.[19] Unfortunately, *GT1* is an unlikely source for Maimonides since it quotes 66;30° for Jerusalem, which brings the distance between the two cities to only 47 minutes. Even recalling a similarity between the alphanumerals for “10” and “50” and thinking of an alternative reading of 78;55° for Raqqa one can arrive at only a 49^m 40^s distance between Raqqa and Jerusalem, while 20 seconds are still wanting.

The Kennedys observed that *GT1* is anonymous, has only 100 localities, and its date is unknown (but probably late) and that the “compiler or copyist was careless and ignorant, for names and numbers are badly garbled.” Such an attestation forces us to look for a more organized and popular tractate in Maimonides’ hands. However, so far our search in this direction has not been successful. The treatises that Maimonides quoted in his works do not seem to contain geographical tables. The first candidate was the book *On Distances* by al-Qâbisî, which Maimonides quoted in his *Guide for the Perplexed*.[20] However, on examining al-Qâbisî’s manuscript at Leiden, Professor Joseph Sadan did not find any geographical tables.

Another possibility could be the *Hakemite Tables* of Ibn Yûnus, which are extant only in fragments. Ibn Yûnus flourished c. 1000 in Cairo, the city where Maimonides wrote his work some 180 years later. However, the Kennedys found in the *Hakemite Tables* 66° for Raqqa’s and 57;50° for Jerusalem’s longitude; the time difference between them falls far short of 50 minutes.[21]

Though the Kennedys claim that all of the other sources in their volume have the digit “3” as the second digit in Raqqa’s longitude, their claim is questionable because of their method - reliance on “common sense” in cases when the writing is problematic:

“Where common sense or a comparison with other sources clearly indicate it, we have not hesitated to read in a missing dot.”[22]

However, this is an *a posteriori* approach, while an unbiased first reading could give another result. Here is an example:

The most authoritative source on early Arabic geographical treatises is Abû 'l-Fidâ (c. 1340).[23] Among Abû 'l-Fidâ's sources, by far the most comprehensive are *Canon* and *Atwâl* - each quotes coordinates of several hundreds cities. Incidentally, only these two give Raqqa's coordinates. When asked to read Raqqa's longitudes in both sources, Joseph Sadan, Professor of Arabic language and literature at Tel-Aviv University, suggested 68;55° for *Atwâl* and 68;50° for *Canon,* and not 63;55° for *Atwâl* and 63;50° for *Canon* as the Kennedys postulated when consulting the same source! Notice that with 56;30° and 56° respectively for Jerusalem's longitude, the former numbers would give the differences of 49^m 40^s and 51^m 20^s respectively - excellent approximations to the value we are after.[24]

The error in assigning Raqqa's longitude could be facilitated by the following circumstances. The 11th century's vague awareness about Raqqa's location was well described by al-Bîrûnî in his book *Kitâb al-Amâkin:*[25]

"The main source of error is their ignorance of the position of Raqqa relative to Baghdad…When I asked one of them where would Raqqa be and to which country does it belong, I found that he had acquired only half the truth about it, and that the full truth could not be gleaned from his scanty knowledge. Yet he refers to it when he uses al-Battânî's zij for calculating the distances between towns…"

Al-Bîrûnî also found that *"Khurasanian calculators took the distances of their towns from Raqqa to be less than their distances from Baghdad by three degrees… So they have made a total error of ten degrees…"* [26]

Following al-Bîrûnî's remarks, it is easy to believe that Arabic geographical sources composed in the 11th century's were greatly confused regarding Raqqa's location and the dot under 3 could be omitted either accidentally or on purpose. It is easy to imagine that the 12th century inherited the 11th century's confusion. Then one can imagine how Maimonides accepted for Raqqa's longitude too high a value - Maimonides was not a professional geographer and never traveled further east than

Jerusalem. With the uncertainty that reigned over Raqqa's location, he could easily have made a wrong choice. Our **conjecture** is that Maimonides used an **average** of the values of Atwal and Canon, read incorrectly.

II. MOLAD VS. MEAN CONJUNCTION

In the first part we explained a discrepancy of about 50 minutes between the epochs of Maimonides and of al-Battânî. Now we explain a difference, close to 27 minutes, between the epoch of al-Battânî and the Jewish *molad*. This explains the total difference of 1 hour and 17 minutes between Maimonides' epoch and the molad.

The solution lies in understanding that the molad is essentially Ptolemy's mean conjunction - a problem we will discuss below - whereas Maimonides (as was convincingly proved by Otto Neugebauer) systematically used al-Battânî's tables, composed c. 882-901.[27]

Then two problems beg to be addressed. One is how far apart the epochs (base point) of the two systems are and the second is whether they go forward synchronously. We show first that the answer to the second question is positive - al-Battânî's mean month is almost identical to Ptolemy's mean (and Jewish calendar) month, which means that Maimonides could consider it permissible to stick to al-Battânî's system. Then we find a difference, close to 27 minutes at Maimonides' epoch, between al-Battânî's conjunction and the Jewish molad. Accounting for a tiny computational error, this explains the rest.

1. The Jewish Calendar Month

It is a well known fact that the length of the Jewish calendar month is equal to the synodic month of Ptolemy: $M = 29^d 12^h 44^m 1^p$. This means that the daily elongation between the mean moon and mean sun[28]

$$\boldsymbol{E = 12;\ 11,\ 26,\ 41,\ 20,\ 18^\circ} \qquad (*)$$

in the Jewish calendar system and in Ptolemy's system is the same.

2. Al-Battânî's Mean Month

Al-Battânî gives the daily speed of the mean sun as *0;59,8,20,46,56,14°*.[29] Unfortunately, he never quotes lunar mean daily motion, though it can be extracted from his tables as *13;10,35,2,7,15,30* .[30] Then the daily motion in elongation is:

$$\mathbf{12;\ 11,\ 26,\ 41,\ 20,\ 19,\ 16\ °} \qquad \textbf{(**)}$$

which disagrees with the Ptolemaic value (*) only in the 5th sexagesimal fraction (less than 0.001 second for a month). This means that for all practical purposes al-Battânî's lunar month is *identical* to Ptolemy's month and to the Jewish calendar month.

3. Al-Battânî's conjunction and Molad

To determine the precise relation between molad and al-Battânî's mean month, let us examine al-Battânî's table with positions of mean sun and moon on March 1, noon, in 20 year intervals. The data range from 931 Era Alexandri (620 AD) to 1611 EA (1300 AD). The table reveals that all al-Battânî's March conjunctions precede the corresponding Molad Nisan by about 27^m.

For example, for March 1, 12:00 of the year 1311 of the Era Alexandri (which is the year 1000 AD), al-Battânî gives the sun's position at *343;55,51°* and moon's position at *247;13,36°,* so elongation at that moment was *96;42,15°*. Dividing the last value by daily motion in elongation (**) *12;11,26,41,19,16°,* one can find that al-Battânî's mean conjunction followed noon of March 1 of the year 1000 by

$$\boldsymbol{7^d\ 22^h\ 22^m\ 55.56^s.}$$

On the other hand, Molad Nisan of the year 1000[31] occurred $8^d\ 22^h\ 49^m\ 43^s$ after the noon preceding March 1, or

$$7^d\ 22^h\ 49^m\ 43.34^s$$

after March 1, noon. The difference between the two quoted numbers is about 26^m 48^s. Let us compose a short table for three of al-Battânî's epochs.

Epoch	***620 AD***	***1000 AD***	***1300 AD***
mean sun at	340;23,37°	343;55,51°	346;43,24°
mean moon at	228;58,05°	247;13,36°	91;06,54°
elongation	111;25,32°	96;42,15°	255;36,30°
conjunction after March 1, noon	$9^d\ 3^h\ 21^m\ 51.087^s$	$7^d\ 22^h\ 22^m\ 55.56^s$	$20^d\ 23^h\ 13^m\ 3.8^s$
Molad Nisan after March 1, noon	$9^d\ 3^h\ 48^m\ 36.666^s$	$7^d\ 22^h\ 49^m\ 43.33^s$	$20^d\ 23^h\ 39^m\ 53.33^s$
difference	$26^m\ 45.58^s$	$26^m\ 47.77^s$	$26^m\ 49.53^s$

Table1. Al-Battânî's conjunctions and Molad Nisan at three epochs.

This means that al-Battânî's month is shorter than the Ptolemaic (Jewish calendar) month by about 1 second in 170 years[32] and that al-Battânî's conjunction in the year 1178 AD preceded the corresponding molad by about $26^m\,49^s$.

4. The remaining 11 seconds

Let us note at this point that performing a "50 minutes" shift, Maimonides made several errors in computation. Neugebauer found that after a correct 50 minute shift of al-Battânî's original values the "mean sun" has to be 0;0,3° later than that of Maimonides, whereas the "mean moon" has to be 0;0,2° earlier than that of Maimonides.[33] This amounts to a total 0;0,5° error in elongation and means that Maimonides made luni-solar conjunction about 10 seconds earlier than he intended.[34] Because of some rounding down, it actually must be 11 seconds.

The answer to Gandz and Neugebauer is that

$$1^h\ 17^m - 11^s = 50^m + 26^m 49^s.$$

The brunt of this formula, albeit trivial, is that the missing $1^h\ 17^m$ consists only of two components and not of four or five, as was suggested by Wiesenberg, and not of three as was suggested by Neugebauer.

Several questions arise from the above analysis.

1. The small difference between Ptolemy's and al-Battânî's mean month raises a question of whether al-Battânî could have used an approximated value of Ptolemy. Could he use, for example, decimals and cut the fraction 1/3 up to the 5th digit after the decimal point?
2. Can a difference in equation of center between Ptolemy's 2;29° vs. al-Battânî's 2;00° alone lead to a half-hour difference in computation of the mean conjunction?
3. Was a computation of the molad part of Halakha c. 1178?

The first two questions are beyond the scope of this paper while the last one we can attempt to answer.

5. Did Maimonides distinguish between molad and mean conjunction?

Maimonides did not make the last step in Neugebauer's computations and the question whether he was aware of the difference between his epoch and molad is debatable. However Neugebauer and Weisenberg took pains to explain the thing as if he was. Let us take for a second this approach seriously and muse what his reaction on discovering the fact could have been.

Comparing molad (presumably fixed for Jerusalem local time) and al-Battânî's epoch for Raqqa he could have noticed that the latter precedes the former, which is impossible if Raqqa lies to the east of Jerusalem! The reason why Maimonides did not address this question could be a lack of any tradition on how molad was fixed. However, he could think in the following direction:

In the literature there is a long-standing and much-debated question of whether molad was fixed as a mean conjunction not for Jerusalem but for another locality, to the east of Jerusalem. The answer, however, depends on another question: which astronomer's mean conjunction became the molad's progenitor? Because the calendar base point, Molad Adam $(6)^d\ 14^h$, is surely a rounded-up value, the progenitor cannot be easily recovered. It could be either the Ptolemaic mean conjunction for Alexandria taken from *Almagest* sometime in 2nd-9th centuries or the mean conjunction computed from observations by some Muslim astronomer (like al-Khwârizmî or Mashallah) in 8-9th century Baghdad or Damascus, further adjusted to Jerusalem's longitude.

There is no indication whatsoever in Maimonides' writings about his awareness of the 8-9th century events. Actually, Maimonides believed [V:3] that the calendar of his own day (and molad in particular) was fixed at the time of Abaye and Rava, Jewish sages of the mid-4th century AD. The only serious astronomical source at that time would have been *Almagest*. So the previous remark could mean that Maimonides believed in the Ptolemaic origin of molad.

Maimonides could know what Solomon Gandz did not - that molad was based on the Ptolemaic mean conjunction, incompatible with that of al-Battânî even after any reasonable geographical correction, and that the reason for the incompatibility, aside from a natural observational error of the central moment of eclipses, could be the different parameters assumed by each astronomer - despite the fact that al-Battânî used the Ptolemaic model and procedure of computations. It does not necessarily mean that in 1178 Maimonides believed in a changing world, but rather the he believed in astronomical parameters which could be established more and more exactly. And in his time, the astronomical community viewed al-Battânî's parameters as more reliable than the Ptolemaic.

Additional Remarks

In his later years, Maimonides made a serious study of the problem of the structure of the world and the motion of the planets. In the *Guide for the Perplexed* of c. 1190 he questioned the validity of the Ptolemaic theory of epicycles and eccenters as being contradictory to the principles of Aristotle. He pronounced the same conclusion over the purely eccentric theory developed by Abû Bakr ibn al-Sâ'igh. At one point Otto Neugebauer asked whether Maimonides understood Eudoxus' theory of the *rotating* *homocentric spheres,* because Maimonides dismissed it as incapable of explaining "retrogradation" of a star, though the theory was created to explain just this phenomenon. Maimonides' precise words are:

> *"And how is it possible to explain retrograde motion without epicycle?"*[35]

True, the epicyclic model is able to explain "retrogradation"[36] as well as changes in the distance of stars from the Earth. Aristotle's theory was able to explain the former, but not the latter. We conclude by echoing Otto Neugebauer:

> *"Whatever the origin of any part of Maimonides's treatise might be, the presentation of the material shows everywhere the great personality of the author and supreme mastery of a subject, worthy of our greatest admiration"*

with a warning that having a "great personality" does not preclude one from simple mistakes (in arithmetic, while fixing the epoch; in methodology, by choosing the wrong geography tractate for references) or ones more grievous (an ambiguous position toward fundamental notions like a definition of molad, and incomplete grasp of the advanced concepts like Eudoxus' theory).

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Appendix. Gauss's Formula for the Molad

"Gauss is always right" (Mathematical parlance)

Gauss's formula for Molad Nisan:[37]

$$M + m = 5.912022885 + 1.5542418 * a + 0.25 * b - 0.003177794 * B,$$

where M is the number of days between March 1 inclusively and Molad Nisan,

m is a fraction of the day counted from noon of the previous day,

B is a C.E. year,

$a=\{\frac{12B+12}{19}\}$ is the is the remainder after division of $(12B + 12)$ by 19;

$b=\{\frac{B}{4}\}$ is the remainder after division of B by 4.

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[1] Maimonides, Moses. *Sanctification of the New Moon*. Code of Maimonides. book III, treatise VIII. Translation by S. Gandz. Introduction by J. Oberman. Commentary by O. Neugebauer. Yale Judaica Series. v. 11. New Haven. Yale University Press. 1952.

[2] Two commentaries on the entire 19 chapters belong to Italian Rabbi Obadia ben David ben Obadia (c. 1340) and a refugee from Spain, Rabbi Levy ibn Haviv (c. 1540).

[3] E.Baneth, 'Maimuni's Neumondsberechnung.' Bericht über die Lehranstalt für die Wissenschaft des Judenthums 16 (1898), 17 (1899), 20 (1902), 21 (1903).

[4] Neugebauer, Otto. 'The Astronomy of Maimonides and its Sources,' (HUCA 22, 1949), pp. 323-63.

[5] Al-Battânî Muhammad ibn Jâbir. *Opus Astronomicum*. Translation from Arabic to Latin and commentary by C.A. Nallino (Milano 1903); reprint (Georg Olms Verlag, Hildesheim-N-Y. 1977).

[6] Gandz, Solomon. *Studies in Hebrew Astronomy and Mathematics.* (Ktav Publishing House, N.Y., 1970), p. 126.

[7] Neugebauer, ibid, p. 329.

[8] Further $1p$ = *1 part* = 1 *chelek* = $3\frac{1}{3}^{s}$.

[9] Gauss' formula (see Appendix with a=12, b=2, B=1178 and M+m=21.31948315) gives Molad Nisan of the year 1178 on $7^{h}\ 40^{m}\ 3.3^{s}$ after midday of March 20, which is exactly 1^{h} 721^{p} after 18:00.

[10] Wiesenberg E., *Code of Maimonides. Appendix.* Yale Judaica Series. v. 11 (New Haven. Yale University Press 1961), p. 579.

[11] The notion of "mean time" seems to be pretty modern (according to Yaqov Loewinger it began with Flamsteed). Al-Battânî's manuscript [op.cit., v. II, pp. 61-4] contains *an equation of time* and had Maimonides decided to make a correction, he could have found there a value of 2;52° for Aries 8 = March 22 (the Vernal equinox in 1178 fell on March 14) and added to the local Jerusalem time of conjunction about 11.5 minutes. Wiesenberg's "4 minutes" came from al- Battânî's equation of time for March 1, which is irrelevant here.

[12]There is, however, a subtle point which baffled all the readers of Maimonides [III:15] -- that he agreed to accept witnesses retroactively, even a week or two after the beginning of the month. See discussion in Wiesenberg [ibid, p. 594].

[13] Neugebauer, ibid., p. 343.

[14] Neugebauer's argument, however, is faulty. Following Nallino he identified Adhar 1 with March 1. Then to find 22 days and 6 hours Neugebauer suggested that Nisan 3 = March 23 which is true but irrelevant, because the beginning of Nisan 3 coincides with 18:00, March 22. This means that either Adhar 1, 1178 had to begin at noon of February 28 (which contradicts our other conclusions below) or that Maimonides made a simple arithmetical mistake, in which case we should take Nisan 4, 0^h (March 23, 18:00) as Maimonides' epoch.

[15] See al-Battânî [ibid., v.2, p. 41, entry 150 and p. 54, entry 273]. The sun covers each degree in the sky for 4 minutes.

[16] Neugebauer, ibid., p. 344.

[17] Neugebauer, ibid., pp. 345-6.

[18] E. S. and M.H. Kennedy *Geographical Coordinates of Localities from Islamic Sources.* (Frankfurt 1987).

[19] Kennedy & Kennedy [ibid., p. 282 and p. xx].

[20] *The Guide for the Perplexed*, II:24; see also Neugebauer [p. 335].

[21] Kennedy & Kennedy [pp. 281, 160]

[22] Ibid, p. .XX.

[23] Abû 'l-Fidâ, *Geographie texte arabe publie par Reinaud et Le Baron Mac Guskin de Slane*. (Paris, 1840).

[24] The Kennedys suggested that *Atwâl* was of "Persian origin" and "independent of *Canon*" but were uncertain about its timing - they placed it c. 900 while Haddad & Kennedy earlier suggested c. 1200. Even with this uncertain timing, *Atwâl's* data closely resembles the tractates mentioned by al- Bîrûnî with erroneous values for Raqqa.

[25]Al-Bîrûnî Abû Reihan, Muhammad ibn Ahmad. *Kitâb al-Amâkin* or *The Determination of the Coordinates of Positions for the Correction of distances between Cities*, transl. from Arabic by Jamil Ali (American Univ. of Beirut, 1967), p. 262.

[26] Ibid., p. 260. One can only guess about the origin of that strange error - probably in the 11th century Raqqa fell into deep obscurity. Eventually al-Bîrûnî (ibid, p. 294) made his own computations and arrived at the longitude 73;40° for Raqqa, close to one of al-Battânî's, and concluded that latter's 73° is a "reliable value."

[27] Neugebauer [ibid., p. 336 and further].

[28] *Ptolemy's Almagest,* IV:3. Tr. & ed. by G.J. Toomer (Princeton Univ. Press, 1998), p. 179.

[29] Al-Battânî [ibid., v.1, p. 42], see also Kennedy, E. S. *'A Survey of Islamic Astronomical Tables.'* Trans. Amer. Phil. Soc., 46, May 1956, pp. 123-177, [p. 156].

[30] One can see that this value, multiplied by 365.25 d and by 680 years, gives the difference 222;8,49° between lunar position 91;6,54° for the year 1300 and 228;58,5° for the year 620. Kennedy*'s Survey* [ibid, p. 156] gives a slightly greater value of 13;10,35,2,7,17,10°.

[31] Gauss' formula (see Appendix with B = 1000, a = 4, b = 0, M+m = 8.951196085) gives Molad Nisan of the year 1000 on $22^h\ 49^m\ 43^s$ after midday of March 7.

[32] Two more epochs were checked at random. The year 800 AD with 342;4,9° for mean sun and 351;18,4° for mean moon makes mean conjunction on 28.77330302 d. after noon of March 1. Molad Nisan 800 AD was 28.79189829 d. after noon of March 1 which gives 26^m 46.6^s difference. The year 1180 AD with 345;36,23° for mean sun and 9;33,35° for mean moon makes mean conjunction on 27.5756159 d. after noon of March 1. Molad Nisan 1180 AD was 27.58433657 d. after noon of March 1 which gives 26^m 48.8^s difference. Both estimates are perfectly consistent with data in Table 1.

[33] Neugebauer, ibid., p. 343. Maimonides also made mistakes in computing positions of the lunar anomaly and line of nodes, but this is unimportant for us at this point.

[34] Division of the distance 0;0,5° by daily motion in elongation 12;11,27° gives about 10 seconds.

[35] *The Guide for the Perplexed*, II: 24.

[36] E.g., if daily motion in anomaly is much greater than daily motion in elongation.

[37] See Gauss's original exposition in Gauss, C.F. *Berechnung Des Juedischen Osterfestes. Monatliche Correspondenz zur Beforderung der Erd- und Himmels-Kunde,* (herausgegeben vom Freiherrn von Zach, Mai 1802) or its elaboration in Sh.B. Burnaby, *Elements of the Jewish and Muhammadan Calendars* (London, 1901), p. 219.