A Talmudic Puzzle and the Jewish Calendar in the Late Third Century

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Introduction: The Talmudic puzzle and Beller's solution

The contemporary Jewish calendar has a 19-year intercalation cycle and is built on the calculation of *molad* (calendrical moments representing lunisolar conjunction) using a mean lunar month. The version we have in our hands, in its essential features, is credited to the 4th century sage, Hillel bar Yehuda Nasi (known also as Hillel II). We know practically nothing about what calendar was in use before the 4th century. Therefore Saadia Gaon c. 922 claimed that the Jewish calendar was given at Mount Sinai (and as such was -- and will be -- always the same). The claim of Sinaitic origin later turned into a powerful myth, reverberated through the centuries to modern times.

Talmud Yerushalmi, written in the end of the 3rd century AD in Eretz Israel, contains a story about the operation of the calendar. There were 24 *watches* of priests who served for one week each, in cyclic order, at the Temple in Jerusalem. At the beginning of each Jubilee year (believed, in various opinions, to occur every 49 years or every 50 years) the watch on duty acquired property that had been bestowed on the Temple by worshippers but not redeemed. According to the Talmud, the choice of 24 by King David as the number of watches represented a "great art" since each watch received its portion once before any watch received it twice. Moreover, this property would not hold if there were only 23 watches. It is written that a certain Rabbi Avahu (*fl. ca*. 300) checked these claims and found them correct. Since it is unreasonable to suppose that R. Avahu had access to many centuries worth of historical records, he must have made his check using a calculation. The problem is: which calculation? For many years, it has been known that the Jewish calendar of today does not have the required property (as we will verify), implying that R. Avahu used a different calendar, or at least a calculation not in exact correspondence with the modern calendar.

In a comparatively recent paper, Eliyahu Beller[3] displayed a solution to this puzzle. Using a basic lunisolar calendar defined by values for the mean lunar month **m** and mean solar year **y**, Beller found that R. Avahu's calculation would only work (with reasonable parameters) if **y** lies in an interval below the correct value[4] for 300AD of $365^d\ 5^h\ 48^m\ 45^s$:

$$365^d\ 5^h\ 9^m\ 45^s < \mathbf{y} < 365^d\ 5^h\ 47^m\ 28^s,$$

together with a value of **m** close to the value $29^d\ 12^h\ 793^p$ that is used in the modern Jewish calendar[5]. Since there is no such year length attested in the ancient literature (Jewish or otherwise), and a value near the upper end of the interval would be the best value for the tropical year in antiquity, this is potentially a discovery of considerable significance.

As supporting evidence, Beller alleged that Maimonides hinted at the existence of this "lost" value when he wrote in his essay on the calendar (Sanctification of the New Moon, X:1):

> Among the Jewish sages who maintain that the solar year is less than 365 1/4 days, there are those who hold that it consists of 365 days, 5 hours, 997 parts and 48 secondary parts -- the secondary part being 1/7 of the (primary) part.

In Beller's opinion, the words "there are those" mean that Maimonides knew of at least one additional value in Jewish tradition, less than 365 1/4 days but different from the other value he gives explicitly.[6]

We now describe Beller's calendrical system. The first year starts at the same moment as the first month. For each integer *k*, the number of calendar months in the first *k* calendar years is the maximum number of whole lunar months that fit into *k* solar years. The actual length of the first *k* calendar years is this number of lunar months, truncated to a whole number of days. Mathematically, if **m** and **y** are expressed in days, the $(k+1)^{\mathrm{st}}$ year begins exactly $[\mathbf{m}[k\mathbf{y}/\mathbf{m}]]$ days after the 1^{st} year begins, where the brackets indicate truncation to integer. With **m** and **y** having plausible values, years have either 12 or 13 months, with 354-355 days or 383-384 days, respectively. These values repeat in a cyclic pattern, but the length of the cycle is long unless **y** and **m** are related by a simple ratio.

A serious problem with Beller's calendrical system is that it is not used in any real calendar known to have existed in ancient times. Nevertheless, it is not totally out of the question that R. Avahu might have used it for his calculations, as we shall see.

What's wrong with Beller's discovery?

We have mentioned that Beller's calendrical system does not correspond to any known calendar. We will return to this point in the next section. Perhaps a more serious objection concerns the length of the year required by Beller's solution. Not only is such a length completely unknown in the historical record, Jewish or otherwise, but there is no record of any values at all less than the astronomically correct value until the advent of Muslim astronomy in the 8^{th} century. As Beller himself noted, not only did the Greeks use a greater value (about 6.5 min. higher than reality) but the Babylonians did as well (about 2 min. higher). There was no other astronomical school at that time known to have produced other accurate values.[7]

Concerning Jewish tradition, Beller's value is seriously in conflict with the evidence. Rabbanim of the 3-4 centuries AD explicitly declared the Julian value $365^d\ 6^h$ to be the truth. Rabbi Avahu is chronologically sandwiched between Shmuel Yarchinai (d. 254) and Abayye (d. 340). The first said (Talmud Bavli, *Erubin* 56ab) that the year is of 365 days and a quarter. The second suggested a special blessing on the sun every 28 years because "the sun comes to the same point on the sky at the same day (Wednesday) as at the time of Creation" (Talmud Bavli, *Berakhot* 59). These two were leaders of their generations. So, assuming no breaks in the Jewish tradition (and the Talmud does not give any hint of such a break at that time), there is no room to suppose that R. Avahu accepted any value other than 365_1/4 days.

Regarding Maimonides' "hint" of a third value, we are not convinced it exists. Alternative explanations for his choice of words are: he knew of a source that mentioned a value less than 365 1/4 days without specifying it; he was just writing cautiously in recognition of his incomplete knowledge of early sources; he wrote "there are those" in order to exclude himself. Concerning the last possibility, we know[8] that Maimonides was a diligent student of Muslim astronomy, and that Muslim astronomers of the 9-12th centuries found a variety of values less than $365^d\ 5^h\ 49^m$.[9]

There is also difficulty with the month value required in Beller's solution, which must be quite close to $29^d\ 12^h\ 793^p$. Despite the antiquity of this value, there is no attestation from earlier than the 9th century[10] of its usage in a Jewish calendrical system. The evidence usually cited, the famous Baraita in Talmudic tractate *Rosh Hashana*, ascribed to Rabban Gamliel of Yavne (d. 116), was a matter of harsh criticism by many students of the calendar (like Slonimsky and Bornstein), who proved that the Baraita suffered multiple emendations.[11]

In summary, Beller's solution appears unlikely when judged against the known history of the period. Moreover, his support from Maimonides is dubious at best. In the following sections, we will present some alternative solutions.

Alternative solutions using Beller's system

The first observation that we wish to make about Beller's conclusions is that he missed a solution. If the first day of the calculation is Thursday, the value $\mathbf{m} = 29^d\ 12^h\ 800^p$ works for $365^d\ 5^h\ 52^m\ 20^s \le \mathbf{y} \le 365^d\ 5^h\ 54^m$, while $\mathbf{m} = 29^d\ 12^h\ 801^p$ works for $365^d\ 5^h\ 53^m \le \mathbf{y} \le 365^d\ 5^h\ 54^m\ 40^s$. Neither this month length nor this year length is attested in ancient Jewish sources, but they are not completely infeasible. The month length differs from the true value by less than 30^s, while the year length is within the range of Babylonian estimates.

We can find a more satisfying solution by removing an assumption in Beller's work. Namely, **Beller assumed that the first watch received its portion in the first year of the calendar**. If R. Avahu was performing a purely theoretical calculation he might have made such an assumption, but there are several reasons he might not have:

1. He might have assumed some tradition about when the Jubilee system began relative to the beginning of the calendar. For example, perhaps the calendar began when it was "given at Sinai" while the Jubilee system began only later. Maimonides (Laws of Shemita and Jubilee) quotes two such traditions: the Jubilee system started in Tishrei either of the year Jews entered Canaan or 14 years after entering, when the tabernacle was established at Shilo and the land was conquered and appropriately distributed.[12] Alternatively, the calendar may have been based on some putative moment of creation like the modern calendar is (though probably the creation reference point was not in vogue in the 3rd century or was different from the point used today.[13]
2. He might have aligned his calculations with the calendar in his own time or observations in his time. For example, he might have worked backwards from a lunar conjunction that he witnessed.

We are also not sure about when the very first watch received its portion. Perhaps it was in the 50th year after the Jubilee system began, as a plain reading of Lev. 25 suggests, when the first Jubilee year occurred. (Curiously, Beller suggests this himself but doesn't use it in his calculations.) Finally, there is a chance that the calendar year started in Nisan but the Jubilee year started in Tishrei.[14]

Whatever the reason R. Avahu had in mind, it is very plausible that he started the calendar and the Jubilee system at different moments. When this possibility is permitted, Beller's calendrical system gives very many additional solutions. One very interesting set of solutions uses the year length 365 1/4 d. (as we have seen, the only year length attested in the Talmud) together with the month length 29^d 12^h 720^p. The latter value appeared in the midrash *Pirqey d_Rabbi Eli'ezer,*[15] as Beller notes. Using these parameters, we get a solution if the first Jubilee year occurred in the k^{th} calendar year and every 50 years from then on, for very many values of k (starting with 2,5,6,9

very many solutions if the calendar year started in Nisan and the Jubilee year started in Tishrei. In either case, there are so many solutions that most of the suggested reasons given above for starting the calendar and the Jubilee system at different moments can be made to "work".

We give one example of such a solution, chosen arbitrarily. If the first year of the calendar started on Sunday, and year 10 was the first Jubilee year, then the sequence of watches receiving their portion is 1, 16, 11, 2, 21, 12, 7, 22, 17, 8, 3, 18, 13, 4, 23, 14, 9, 24, 19, 10, 5, 20, 15, 6. All the Jubilee years in this solution start on Sunday, Monday or Tuesday.

To make this solution plausible, it remains to explain why R. Avahu may have used Beller's calendrical system even though it was probably never used in a real calendar. We offer one possibility. R. Avahu probably believed, no doubt correctly, that until their dispersal the Jews set their months and years by direct observation of the moon and seasons. Therefore, he might well have chosen to avoid the artificial structure of whatever calendar was official in his time and instead calculate the years and months himself using the best values he knew for the mean lengths of each.

In summary, by generalizing Beller's calculations just a little, we have found plausible solutions to the Talmudic puzzle that do not demand astronomical constants unsupported in the historical record.

Another solution: the 8-year calendar

We now find another solution to the Talmudic puzzle, this time using a historical calendar. There is a historically documented statement of Julius Africanus[16] (c. 220 AD) that an 8-year calendar was in use by Jews at the beginning of the 3rd century. Exactly when it was replaced by a 19-year calendar is unknown, but it is at least plausible (if unlikely) that it was still in use in R. Avahu's time. In this calendar there are 5 years of 354 days and 3 years of 384 days in each 8-year cycle. The mean year of this calendar is 365 1/4 days, and the mean month is approximately $29^d\ 12^h\ 393^p$.

Assuming that the intercalated (384-day) years were never adjacent, there is only one possible intercalation pattern up to rotation, but (for the reasons expanded in the previous section) we need to consider that the first Jubilee year might have occurred at any point in the cycle. We also need to consider that the first day of the first year might have been on any day of the week. Finally, Jubilee years may have occurred every 49 years or every 50 years. This gives $8 \times 7 \times 2 = 112$ possibilities. Exactly four of these give a solution to the Talmudic puzzle, each of them requiring a Jubilee cycle of 50 years. The first day of the first Jubilee year can be Tuesday or Wednesday, and the first intercalated years can be years 1,3,6 or years 1,4,6 (counting the first Jubilee year as year 1). As with our discussion in the previous session, there are several reasons the calendar used by R. Avahu may have been aligned to the Jubilee system in this apparently irregular manner.

There are also solutions for which the calendar begins with Nisan and the Jubilee year begins with Tishrei, again using a 50-year period.

A problem with the 8-year calendar is that it cannot be used for very long before the inaccuracy in the mean month (about 1.5 days in each 8-year cycle) becomes unacceptable. It is most unlikely that it had been in use for so long in R. Avahu's time that he could have believed it had been in use since the beginning of the Jubilee system. However, if the inaccuracy in the calendar had not yet become known, he might have imagined it was accurate enough to approximate the earlier observational calendar when extrapolated backwards from a lunar conjunction in his time.

One way to correct the inaccuracy of the mean month in this system, at the expense of making the mean year even less accurate, would be to insert 3 extra days in each two 8-year cycles. However, none of the possible regular patterns for inserting the extra days leads to a solution. This is still true using the Nisan/Tishrei option mentioned above.

In summary, the 8-year calendar, which might have been still in use in R. Avahu's time, provides another possible solution to the Talmudic puzzle. However, we feel that the solution in the previous section is more likely.

Possibly a 30-year calendar?

In four places[17] Talmud Bavli mentions the following passage:

> And others say: from Atzeret to Atzeret and from Rosh Hashana to Rosh Hashana - 4 days, though in an intercalary year - 5 days.

This suggests a calendar, known as the "theory of others", about which there is very little direct knowledge. Recently Ari Belenkiy[18] showed that, if "intercalary year" was interpreted as "Julian leap year", the comment can be taken as indicating a calendar built on top of the Julian calendar. In addition to having years of 12 or 13 months in the usual fashion of lunisolar calendars, there are leap-days added every four years. At least two such calendars are known from about the required time period, using intercalation cycles of 19-years and 30-years. In the same 2002 paper Belenkiy proposed the 19-year calendar as the "theory of others", and we will deal with it in the next section. In this section we will consider the 30-year calendar, since it provides a feasible (though less satisfactory) solution to our puzzle, as well as being an alternative possibility for the "theory of others".

A calendar with a 30-cycle was used for a time by the Eastern (Antiochean) Church, as revealed by the Sardica Document (c. 343).[19] That document shows the 30-year cycle of intercalation, aligned with the first 16 years of the contemporary Jewish sequence. It is not possible to be sure from only 16 years whether the Jewish sequence also had a 30-cycle, as the usual 19-year pattern also matches, but we will consider the possibility here. In each 30-year cycle, there are 19 years of 354 days (355 in a leap year) and 11 years of 384 days (385 in a leap year). The mean year is 365 1/4 days and the mean month is close to $29^d\ 12^h\ 908^p$.

In considering R. Avahu's calculation, we need to consider that the first Jubilee year may have occurred at any point in the 30-year cycle, and also at any point in the 4-year Julian cycle. In addition, the first day may have been any day of the week and the Jubilee cycle might have had 49 or 50 years. This gives 1680 possibilities altogether, but all of them lead to repetitions in the first 24 watches. We also considered that the calendar might have started in Nisan but the Jubilee year in Tishrei, with the same negative result. However, there are a few very near misses for a Jubilee cycle of 49 years that caught our attention. For example, the following sequence of watches is achieved if the intercalary years starting at the first Jubilee year (called year 1) are years 2,5,8,10,13,16,18,21,24,27,29, and the first leap year is year 2. The number after the slash is the day of the week at the start of the Jubilee year (0 for Sunday):

1/3, 14/2, 3/2, 16/1, 5/0, 18/6, 7/6, 19/5, 8/4, 21/3, 10/3, 23/2,

12/1, 20/5, 9/5, 22/4, 11/3, 24/2, 13/2, 2/1, 15/0, 4/6, 17/6, 5/5.

The only "error" is the repetition of watch 5 in the last step, but this can be corrected by delaying the start of the last Jubilee year by 2 days. Of course, the starts of most of the other Jubilee years can also be delayed by a few days provided they stay in the same week. Rules (*dekhiyot*) imposing such small delays exist in the modern Jewish calendar, but it is not known exactly when they were introduced. Nor is it known whether they were always the same as now, but in any case we don't know enough about the actual location of the solution in real time to apply *dekhiyot* even if we knew what those rules were.[20]

In conclusion, it is just possible that some system of *dekhiyot* used in conjunction with a 30-year calendar was the basis of R. Avahu's calculation.

Searching for solutions with 19-year calendars

In this section we describe our unsuccessful search for a solution to the puzzle using a calendar with a 19-year cycle. Two such calendars were considered - the modern Jewish calendar and a 19-year calendar that may have been the "theory of others" (although that honour might well belong to the 30-year calendar mentioned in the previous section).

In the case of both calendars, we will allow for the possibility of slight variations in the definitions that affected the starts of years by a few days. To achieve this, we define a **standard form** of the calendar (which for the modern Jewish calendar is its modern form), but look for solutions using any calendars which are nearby in the following sense: in any interval of years of up to 23 full Jubilee cycles, the number of days differs by at most 7 from the number of days in the standard form. For example, this allows for a calendar the same as the modern Jewish calendar except that there are no *dekhiyot* (discussed in the previous section). In the case of the 19-year possible "theory of others", this device covers our uncertainty (discussed below) about how leap days and *saltus lunae* were applied and allows for existence of a few *dekhiyot.*

We remind the reader of the essence of the 19-year calendar that might be the "theory of others".[21] This system is equivalent to the "epact" theory also used by the 4th century Church. This system was attached to the Julian calendar. Every year all Jewish holidays were shifted by 11 days down the calendar and in the intercalary years (7 per 19-year cycle), shifted up by 30 days in addition. Doing that, one can easily discover that new cycle will start by one day later so the Church once in a cycle (in the last, 19th, year) used a downward shift of 12 days instead of 11 days. These shifts got the name "*saltus lunae*". We take the "epact" system to be the standard form of this calendar. We don't know the details of the system actually used by the Jews, as the Talmud is silent on them. For example, there are reasons to suspect the *saltus lunae* were applied to Julian leap years (canceling the leap day), or by some more elaborate scheme, rather that at a fixed point in the cycle. However, our allowance of a 7-day divergence from the standard form is enough to cover most such possibilities.

Despite the large number of possible calendars included in our search, none of them satisfy the requirements of R. Avahu's calculation. We also tried years starting in Nisan instead of Tishrei, with the same lack of success. There is an outside possibility, however. If R. Avahu believed in a year length slightly different from the 365 1/4-day average of the "theory of others", he might have considered adjusting the intercalation pattern of that system at some points in his 23-Jubilee calculation in compensation. There are in fact solutions using such occasional adjustments, but their detailed justification remains obscure.

In conclusion, we did not manage to find a convincing solution using a known 19-year calendar.

Conclusions

We have demonstrated that the puzzle presented by the Talmud Yerushalmi has many possible solutions other than the one found by Beller. Some of the solutions use only well-attested month and year lengths, so there is no need to postulate others. In addition, we have demonstrated that neither the modern 19-year calendar, nor another historical 19-year calendar, is even close to providing a solution. Thus, the evidence of Talmud Yerushalmi is incompatible with traditional beliefs of the great antiquity of the modern calendar.

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[3] Eliyahu **Beller**, 'A Newly-Discovered Ancient Value for the Length of the Year' (*Arch. Hist.Exact Sci.,* 52, 1998, pp. 91-98.)

[4] The slightly larger value Beller gave for 300AD is incorrect, as it measures the ancient year in terms of the modern day. Nevertheless, the difference between the ancient and modern astronomical parameters is too small to make an appreciable difference anywhere in this work.

[5] The superscript *p* refers to *parts* (*halakim*), of which there are 1080 in an hour. The value $29^h\ 12^m\ 793^p$ is only a fraction of a part different from the astronomically correct value. This remarkably accurate value has a long history, being known to the Babylonians at least by 250 BC and to the Greeks by c. 140 BC. Later it appeared in Ptolemy's famous *Almagest*.

[6] The second value given explicitly by Maimonides, $376^d\ 5^h\ 55^m\ 25^s$ in modern units, is related to the month value $29^d\ 12^h\ 793^p$ (see footnote 5) by the Metonic cycle 235**m**=19**y**. For some time in the 10th century, it was thought by Jewish writers to be exact. Later tradition attributed the value to Rav Adda of the 3rd century, a disciple of Shmuel, but historical support for this is lacking.

[7] The Hindu astronomers were able to measure the value of the sidereal year, which is greater than the Julian year.

[8] See Otto **Neugebauer**, 'The Astronomy of Maimonides and its Sources', *HUCA*, xxii, 1949.

[9] One can make a picture of that epoch reading E.S. **Kennedy**, 'A Survey of Islamic Astronomical Tables'. *Trans. Amer. Phil. Soc.*, 46, May 1956, pp. 123-177. All these values were below the present-day value and the reason is known: after finding the position of the Vernal equinox they compared it with the one found by Ptolemy which some reason was 28 hours later than should have been, see, e.g., Robert **Newton**, *The Crime of Clavdius Ptolemy (*The John Hopkins University Press: Baltimore and London 1978.)

[10] al-Khwarizmi (823/824), see E.S. Kennedy, 'Al-Khwarizmi on the Jewish Calendar'. *Scripta Matematica*, 27, 1964, pp. 55-59.

[11] See Ch. Y. **Bornstein**, *Makhloket bein Rav Saadyah Gaon uBen Meir* (Warsaw 1904), or a brief summary of their arguments in Ari **Belenkiy,** 'Sod Haibbur: Shalosh Shitot B'luach ha-Ivri B'meot ha-Rishonot L'sphira', in the *Proceedings of the 11th Conference on the History of Judea and Samaria* (Ariel 2002, pp. 275-86).

[12] For the second option see the Code of Maimonides, *Hilkot Shemitta* and *Yovel*, ch. X. Also see the discussion in Edgar **Frank**, *Talmudic and Rabbinical Chronology* (Philipp Feldheim: N.Y. 1956), p. 71.

[13] See, eg, A. Belenkiy, 'Sod Haibbur'.

[14] While Jewish sages of Babylonia were in favour of reckoning time from Tishrei, Jewish sages from Eretz Israel (Land of Israel) always reckoned from Nisan. This goes back to the differences of how different peoples of the Middle East reckoned the beginning of the Selecuid Era, see, eg, Frank, op.cit., p. 30.

[15] Ari Belenkiy in 'Sod Haibbur' argues that this value should be attributed to Shmuel Yarchinai (d. 254) and might represent the only original Jewish contribution to the ancient astronomy.

[16] Emil **Schurer**, *The History of the Jewish People in the Age of Jesus Christ* (T&T Clark: N.Y. 1973) Appendix 3. Sacha **Stern** in *Calendar and Community* (Oxford Univ. Press: Oxford 2001) suggested that Julius Africanus was referring to the time of Nehemia (c. 350 BC) while Ari Belenkiy in 'Sod Haibbur' argued that the Church father was describing a contemporary calendar. Either case is possible. In the first case R. Avahu could think in historical terms, extrapolating the time of Nehemia back to King David's time.

[17] *Rosh Hashana* 6b and 20a, *Sukka* 54b, *Shabbat* 87b. "Rosh Hashana" means "New Year," "Atzeret" means "Shavuot" [Pentecost], celebrated 50 days after Passover.

[18] A. Belenkiy, 'Sod Haibbur'.

[19] See, eg, Stern, *Calendar and Community, pp.*124-132.

[20] If R. Avahu began his computations in the time of the 1st Temple or the time of King David (c. 840 BC by the rabbinical chronology, see Frank, op.cit., p.11), it is even plausible that he might apply *dekhiyot* only at the last part of his 1150 year calculation. Only the last Jubilee year would have been still in the future, and the first known discussions on *dekhiyot* recorded in the Talmud (Bavli*: Rosh Hashana* 20a, Yerushalmi*: Avoda Zara* 39b, *Megilla* 70b, *Sukka* 54b) are dated c. 280. The implication that the system of watches had been established with knowledge of the future would have caused little disquiet for someone who believed that King David and Solomon were great prophets.

[21] A. Belenkiy, 'Sod Haibbur'.