Basic Probability Equations

General

Mutually exclusive:

if $E \cap F = EF = \emptyset$ then E,F are called *mutually exclusive*.

Conditional probability:

$$P(E|F) = \frac{P(EF)}{P(F)} \qquad \text{or} \qquad P(EF) = P(E|F)P(F)$$

Chaining rule:

$$P(\bigcap_{i=1}^{n} A_i) = P(A_1A_2 \ldots A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1A_2) \cdots P(A_n|A_1A_2 \ldots A_{n-1})$$

Unification:

$$P(\bigcup_{i=0}^{n}(E_i)) = \sum_i P(E_i) - \sum_{i<j} P(E_iE_j) + \sum_{i<j<k} P(E_iE_jE_k) + \cdots + (-1)^{n+1}P(E_1E_2E_3 \ldots E_n)$$

Independent events:

E and F are *independent* if $P(EF) = P(E)P(F) \Leftrightarrow P(E|F) = P(E)$

$E_1 \ldots E_n$ are independent if for every subset i $E_{i_1} \ldots E_{i_k}$ of them $P(E_{i_1} \ldots E_{i_k}) = \prod_{j=1}^{k} P(E_{i_j})$

Bayes' formula:

$$P(F|E) = \frac{P(E|F)P(F)}{P(E)}$$

Marginal probability:

$$P(E) = \sum_{i=1}^{n} P(E|F_i)P(F_i) \qquad \text{for any } F_1 \ldots F_n \text{ that are mutually exclusive}$$

Bayes and Marginal:

$$P(f|E) = \frac{P(E|f)P(f)}{\sum_{i=1}^{n} P(E|F_i)P(F_i)}$$

Joint probability of r.v. (random variable):

$$P_X(X) = \sum_y p(x,y)$$

Expectation:

$$E[X] = \sum_{x \in X} xp(x)$$

$$E[aX+b] = aE[X]+b \qquad \text{and} \qquad E[X+Y] = E[X]+E[Y]$$

$$\text{therefore, for any} \qquad a,b>0 \qquad E[aX+bY] = aE[X]+bE[Y]$$

Expectation of a function of a r.v.:

$$E[g(X)] = \sum_{x \in X} g(x)p_X(x)$$

Expectation of jointly distributed r.v.:

$$E[g(X,Y)] = \sum_y \sum_x g(x,y)p(x,y)$$

For independent r.v.:

$$E[g(X)h(Y)] = E[g(X)]E[h(Y)]$$

Conditional expectation:

$$\text{for any r.v X,Y:} \qquad E[X] = E_Y[E_X[X|Y]] = \sum_y E_X[X|Y=y]P(Y=y)$$

Variance:

$$Var(X) = E[(X-E[X])^2] = E[X^2]-(E[X])^2$$

$$Var(X+Y) = Var(X)+Var(Y)+2Cov(X,Y)$$

Covariance:

$$Cov(X,Y) = E[(X-E[X])(Y-E[Y])] = E[XY]-E[X]E[Y]$$

(if X,Y are independent then $Cov(X,Y)=0$)

Types of Random variables

Binomial: $X \sim B(n,p)$

$$P(X=k) = \binom{n}{k}p^k(1-p)^{n-k} \qquad E(X)=np \qquad Var(X)=np(1-p)$$

Geometric: $X \sim G(p)$

$$P(X=k) = (1-p)^{k-1}p \qquad E(X)=\frac{1}{p} \qquad Var(X)=\frac{1-p}{p^2}$$

Poisson: $X \sim POISS(\lambda)$

$$P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!} \qquad E(X)=\lambda \qquad Var(X)=\lambda$$